IJCDCG3 2023

The 25th INDONESIA-JAPAN CONFERENCE ON DISCRETE AND COMPUTATIONAL GEOMETRY, GRAPHS, AND GAMES

Courtyard by Marriott Bali
Nusa Dua Resort, Bali, Indonesia

22-24 September 2023

ABSTRACT BOOK

Organized by

The Combinatorial Mathematics Research Group
at Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung,
The Indonesian Combinatorial Society (InaCombS), and
Center for Research Collaboration in Graph Theory and Combinatorics, Indonesia

The series of the Japan Conference on Discrete and Computational Geometry, Graphs, and Games (JCDCG3)
has been held annually since 1997, except for 2008 and 2020.

The 25th Indonesia-Japan Conference on Discrete and Computational Geometry, Graphs, and Games

Courtyard by Marriott Bali Nusa Dua Resort, Bali, Indonesia
22-24 September 2023

Invited Speakers

Hilda Assiyatun	Institut Teknologi Bandung, Indonesia
Erik Demaine	Massachusetts Institute of Technology, USA
Miquel Angel Fiol	Universitat Politècnica de Catalunya, Spain
Stefan Langerman	Université Libre de Bruxelles, Belgium
Kenta Ozeki	Yokohama National University, Japan
János Pach	Rényi Institute of Mathematics, Hungary
Nick Wormald	Monash University, Australia

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Outline of Program

Friday, 22nd of September

08.00 - 08.30	Registration
08.30 - 09.00	Opening
09.00 - 09.45	Invited Talk 1 by Erik D. Demaine
09.45 - 10.30	Invited Talk 2 by Nick Wormald
10.30 - 11.00	Coffee Break
11.00 - 11.45	Invited Talk 3 by Hilda Assiyatun
11.45 - 14.00	Lunch
14.00 - 14.45	Invited Talk 4 by Miquel Angel Fiol
14.45 - 16.00	Onsite Parallel Session
16.00 - 16.15	Coffee Break
16.15 - 17.30	Onsite Parallel Session

Saturday, 23rd of September

09.00 - 09.45	Invited Talk 5 by Stefan Langerman
09.45 - 10.30	Invited Talk 6 by Kenta Ozeki
10.30 - 11.00	Coffee Break
11.00 - 12.45	Onsite Parallel Session
12.45 - 14.00	Lunch
14.00 - 14.45	Invited Talk 7 by János Pach
14.45 - 16.00	Onsite Parallel Session
16.00 - 16.15	Coffee Break
16.15 - 17.30	Online Parallel Session
18.45 - 21.00	Conference Dinner

Sunday, 24th of September

08.00 - 09.00	Online Parallel Session
09.00 - 11.00	Onsite Parallel Session
11.00 - 11.45	Closing
11.45 - 12.45	Lunch

Preface

Welcome to Bali! Welcome to IJCDCG^3 2023!

The Indonesia-Japan Conference on Discrete and Computational Geometry, Graphs, and Games (IJCDCG^3) 2023 is the 25th edition of the series of the Japan Conference on Discrete and Computational Geometry, Graphs, and Games (JCDCG^3) has been held annually since 1997, except for 2008 and 2020. This conference is organized by the Combinatorial Mathematics Research Group at the Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, and supported by the Indonesian Combinatorial Society (InaCombS) and Tokyo University of Science.

As in the previous editions of JCDCG^3, the conference's topics are, but are not restricted to, Discrete Geometry, Computational Geometry, Graph Theory, Graph Algorithms, and Complexity and Winning Strategies of Puzzles and Games. This year, the conference is conducted in hybrid mode, with 93 participants from 16 countries: Australia, Belgium, Catalonia, China, France, Hungary, India, Iran, Philippines, South Korea, Thailand, UK, USA, Vietnam, in addition to Indonesia and Japan.

We want to express our gratitude to all the invited speakers: Hilda Assiyatun (Institut Teknologi Bandung, Indonesia), Erik Demaine (Massachusetts Institute of Technology, USA), Miquel Angel Fiol (Universitat Politècnica de Catalunya, Spain), Stefan Langerman (Université Libre de Bruxelles, Belgium), Kenta Ozeki (Yokohama National University, Japan), János Pach (Rényi Institute of Mathematics, Hungary), and Nick Wormald (Monash University, Australia), who are willing to share the knowledge in this conference. We would also like to thank Institut Teknologi Bandung and Tokyo University of Science for their generous support towards the organization of the conference.

We hope all participants enjoy Bali, exchange knowledge, and initiate fruitful collaboration during the conference.

Dr. Rinovia Simanjuntak
Chair of the Organizing Committee

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Invited Talks

RAMSEY-MINIMAL GRAPHS FOR COMBINATIONS CONTAINING MATCHINGS OR STARS

Hilda Assiyatun

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Institut Teknologi Bandung, Indonesia

Abstract

Let F, G, and H be simple graphs. The notation $F \rightarrow (G, H)$ means that for any red-blue coloring on the edges of F, there exists either a red copy of G or a blue copy of H. If $F \rightarrow (G, H)$ then F is called a Ramsey graph for (G, H). In addition, if F satisfies that $F - e \not\rightarrow (G, H)$ for any edge e of F, then F is called a Ramsey (G, H)-minimal graph. The set of all Ramsey (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$. The study on the Ramsey minimal graphs was initiated by Burr, Erdős, and Lovász in 1976. In this talk, I will discuss some recent progress in Ramsey (G, H)-minimal graphs, particularly construction methods for (G, H) containing matchings or stars.

New Adventures in Puzzle Fonts

Erik D. Demaine

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Massachusetts Institute of Technology

(This talk is based on joint work with Martin L. Demaine and others.)

Abstract

What if the *way* we write text, not just the text itself, expresses the mathematics we are writing about? Even further, what if reading the text requires engaging in mathematical puzzles? In this talk, I will show several newer mathematical and puzzle fonts that explore these questions, from pencil-and-paper puzzles to computational origami to video games to integer sequences. Figures 1 and 2 show some examples. I will also describe some of our latest explorations into using mathematical/puzzle fonts as tools to design algorithmic art.

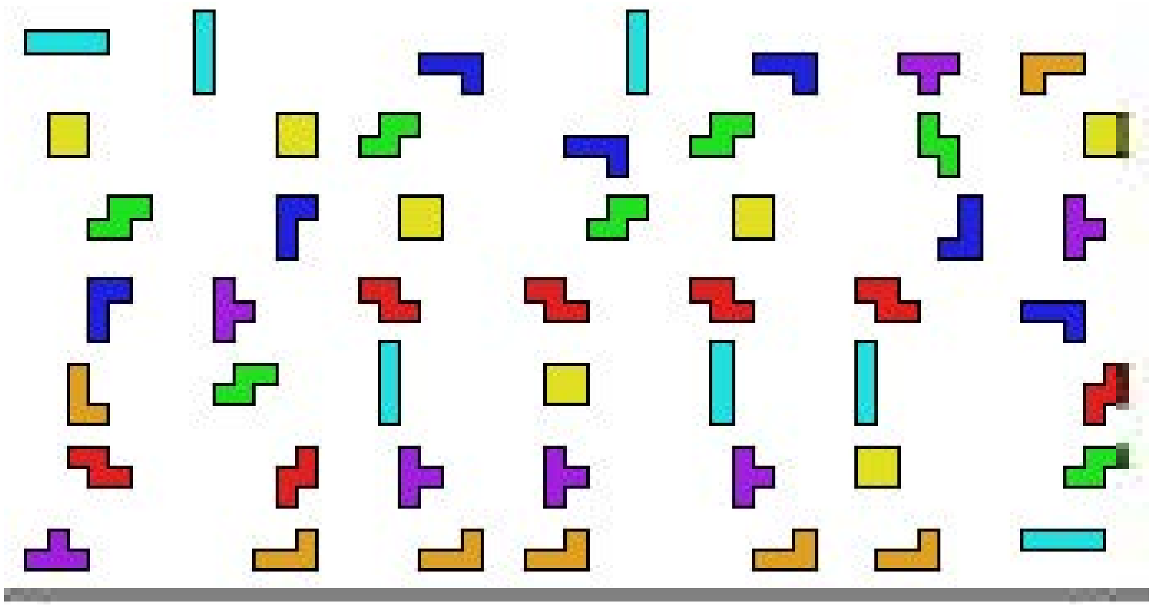

Figure 1: What message results if all the Tetris pieces fall straight down until they are supported by another piece or the ground? To see the solution, visit https://erikdemaine.org/fonts/tetris/?text=%3A%3B4548b&rot=1&puzzle=1

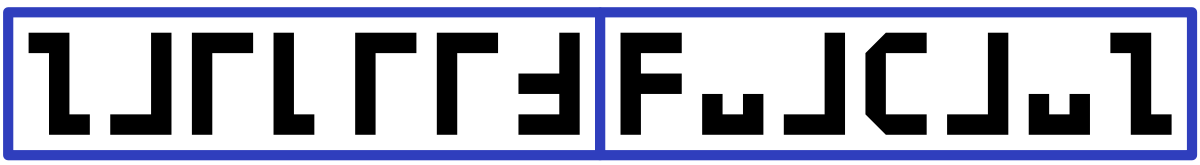

Figure 2: What message results if you fold this figure in half along the vertical blue line? To see the solution, visit https://erikdemaine.org/fonts/silhouette/?text=%3A%3B4548b&rot=1 and click on the figure.

Token graphs of Cayley graphs as lifts

Miquel Angel Fiol

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Universitat Politècnica de Catalunya, Spain

(This talk is based on joint work with C. Dalfó, S. Pavlíková, and J. Širáň.)

Abstract

In this talk, we describe a general method to represent k-token graphs of Cayley graphs as lifts of voltage graphs. In particular, this allows us to construct circulant graphs and Johnson graphs as lift graphs on cyclic groups. As an application of the method, we derive the spectra of the considered token graphs. The method can also be applied for dealing with other matrices, such as the Laplacian or signless Laplacian, and to construct token digraphs of Cayley digraphs.

Tiling Algorithms

Stefan Langerman

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Département d'Informatique, Université Libre de Bruxelles, Belgium

Abstract

A *tiling* is a covering of the plane with copies of one or more geometric shapes (*tiles*) without gaps or overlaps.

Tilings have been used since ancient times in construction, design and art, to create beautiful and mesmerizing patterns. But how does a designer go about creating a set of tiles? And once these tiles are at hand, how does one assemble them to cover the plane?

In this talk, I will go over the history of tiling algorithms, outline exciting developments from recent years, and highlight some of my favorite open problems on the subject.

Spanning trees in star-free graphs

Kenta Ozeki

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Yokohama National University, Japan

Abstract

Related to Matthews–Sumner Conjecture, several sufficient conditions for the Hamiltonicity of star-free graphs have been proven, and recently, those have been extended to the existence of spanning trees with particular conditions, such as spanning trees with bounded number of leaves and those with bounded maximum degree. In this talk, I give some recent results on the topic.

Balls and holes

János Pach

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Rényi Institute of Mathematics, Hungary

Abstract

TBA.

The random graph d-process

Nick Wormald

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Abstract

A graph d-process starts with an empty graph on n vertices, and adds one edge at each time step, chosen uniformly at random from the remaining non-edges subject to the constraint that no vertex degree may exceed d. The final graph must be d-regular except for at most d vertices of lower degree. Erdős posed the question of finding the distribution of the degree sequence of the vertices in the final graph. Once upon a time, Andrzej Rucinski and I showed, using a martingale argument, that asymptotically almost surely the final graph is regular if nd is even, and has just one vertex of degree less than d if nd is odd. About two decades later, we announced new approach to analysing this process which allows us to obtain much more accurate answers to a number of questions, such as: What is the degree distribution at some point the process? How unlikely is it that the final graph is not regular when nd is even? When does the last vertex of degree 0 disappear?

I will discuss recent developments in the study of the d-process, focussing on this joint work with Rucinski, which has undergone simplifications in the intervening time.

Contributed Talks - Discrete Geometry

Maximum metric embeddings

Robert D. Barish

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The University of Tokyo

(This talk is based on joint work with Tetsuo Shibuya.)

Abstract

Provided a non-negative, hollow, and symmetric distance matrix $\mathcal{D}$ with at most q unique pairwise distances (specified to some finite precision), we consider variations on the problem of finding a maximum set of points consistent with $\mathcal{D}$ that satisfy the triangle inequality and can be embedded in a given metric space. In particular, we show that deciding the existence of a set of at least k points admitting a metric embedding is $W[1]$-complete for parameter k, that counting the number of such sets is $\#W[1]$-complete for parameter k (see Flum & Grohe [1] concerning the $\#W[i]$ hierarchy of complexity classes), and that approximating the largest possible cardinality set of points under this embedding constraint is at least APX-hard. In addition, we show that these results hold $\forall q \geq 2$, even in if we require embeddings to be in ultra-metric spaces satisfying the strong triangle inequality. Finally, we examine the applicability of well-known dichotomy theorems for Boolean Constraint Satisfaction Problems (CSPs) to these metric embedding problems.

References

[1] J. Flum and M. Grohe: The parameterized complexity of counting problems. *SIAM J. Comput.* **33**(4); 2004; DOI: 10.1137/S0097539703427203.

Enumeration on the Convex Composition of Polygons in Lucky Puzzle

Thanatnon Keelapang, Supanut Chaidee, Piyashat Sripratak

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Abstract

Similar to Tangram, Lucky Puzzle consists of seven polygons combined to be a rectangle with ratio 4 : 5 that the player has to arrange to form the desired shape. One of the fascinating difficulties is considering the composition of the polygons to be a convex polygon. In this investigation, we present a criterion to enumerate the possible convex forms that can be created from the 7 polygons that make up Lucky Puzzle. In addition, we gave the lemmas related to geometric properties to filter some impossible solutions. We also showed that for some convex polygons that satisfy the requirements in the lemmas, they cannot be formed by any compositions of 7 Lucky Puzzle polygons. We observed that there are 49 solutions that satisfy the composable requirement of being a convex polygon. Currently, we can identify 25 patterns that can be combined to form a convex polygon, while we expect that the other 24 patterns are impossible to combine. We are currently working on the verification of the non-composabilities of these patterns.

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[1] Beelen, T. G. J. (2017). Finding all convex tangrams. (CASA-report; Vol. 1702). Eindhoven: Technische Universiteit Eindhoven.

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[3] Demaine, E. D., Korman, M., Ku, J. S., Mitchell, J. S., Otachi, Y., van Renssen, A., ... & Uno, Y. (2020). Symmetric assembly puzzles are hard, beyond a few pieces. Computational Geometry, 90, 101648.

[4] Wang, F. T., & Hsiung, C. C. (1942). A Theorem on the Tangram. The American Mathematical Monthly, 49(9), 596-599.

Runnin' in the Rain Formula

Hiro Ito

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Abstract

Sometimes we hear a folk belief that *the amount of wetness is the same whether you walk or run in the rain.* We can find that this is a superstition by using an easy thought experiment: it is clear that running a very short distance (e.g., one meter) in heavy rain and walking very very slowly will result in a distinctly different amount of wetness.

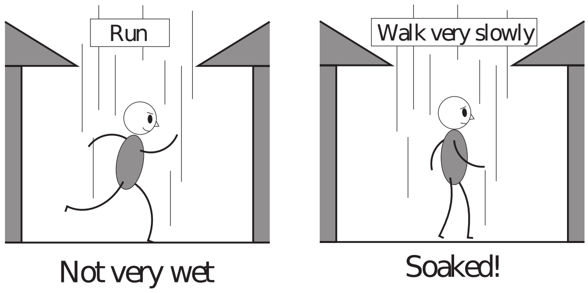

Figure 1: Running does not make you so wet, but walking very slowly makes you soaked.

It will not difficult to derive a formula approximating the wet volume by introducing some appropriate assumptions and using parameters expressing conditions, e.g., the shape of the person, walking speed, the velocity of the rain, etc. We were, however, worried about that the folk belief asserts "the same," and we considered why such a folk belief appeared and tried to derive an approximation formula expressing the relation. As a result we obtained the following formula.

> **Runnin' in the Rain Formula:** The wet amount of an average sized person walking at normal speed in the rain (not drizzle) without strong wind is normalized as one. Then the wet amount of him/her running/walking x times faster in the same condition is approximated as follows.

$$\mathrm{RiR}(x) = \frac{1}{2}\left(1 + \frac{1}{x}\right). \tag{1}$$

From this formula, we can get the following observations.

Observation 1 No matter how fast you run (even at the speed of light), you will only get half as wet at most. (Thus we may say "the amount of wetness is *not so different* whether you walk or run in the rain.")

Observation 2 From $\lim_{x\to 0} \mathrm{RiR}(x) = \infty$, you will get wet as much as you want by walking slowly.

We firstly showed this formula in the 14th research meeting on combinatorial games and puzzles in 2019 (in Japanese) [2] and it was presented in a Japanese magazine [3] and a media [5]. However, we have not introduced this formula in Eanglish yet. This is the first time to present it in English.

Certainly many results on this topic have been obtained (e.g. see [1, 4]). Any result, however, has not derived simple formulae as ours (1). The importance of the Runnin' in the Rain Formula is that *the relation can be approximated in such a simple formula in the average case*. We consider that this has not been given so far.

In this talk we will show how this formula was obtained and moreover our speculation on why the folk belief has appeared.

References

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The maximum volume of lattice corner 3-simplices with fixed number of interior lattice points

Takayasu Kuwata

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Tokai University

(This talk is based on joint work with Shoto Ishida.)

Abstract

A convex polytope $P \subset \mathbb{R}^d$ is called a lattice $d-$polytope if all vertices of P belong to $\mathbb{Z}^d$ and $\dim(P) = d$.

For $k \in \mathbb{Z}_{\geq 0}$, let $\mathcal{P}^d(k)$ be the family of all lattice $d-$polytopes in $\mathbb{R}^d$ which have exactly k interior lattice points, let $\mathcal{S}^d(k) := \{S \in \mathcal{P}^d(k) : S \text{ is a simplex}\}$, and let $\mathcal{CS}^d(k) := \{\Delta \in \mathcal{S}^d(k) : \Delta \text{ is unimodular equivalent to a corner lattice simplex}\}$, where Δ is called a corner $d-$simplex if it is defined by a system of inequalities like $\dfrac{x_1}{a_1} + \dfrac{x_2}{a_2} + \cdots + \dfrac{x_d}{a_d} \leq 1$, $x_i \geq 0 (i = 1, 2, \cdots, d)$ in $\mathbb{R}^d$.

Fix $d \geq 1$, $k \geq 0$. We consider the upper bound of $\{\mathrm{Vol}(P) : P \in \mathcal{P}^d(k)\}$, where $\mathrm{Vol}(P)$ expresses the normalized volume of P which is $d!$ times of the usual volume $\mathrm{vol}(P)$. Since $\sup\{\mathrm{Vol}(P) : P \in \mathcal{P}^d(0)\} = +\infty \, (d \geq 1)$ holds, we assume $k \geq 1$.

In case $d = 2$, Pick's theorem and Scott's theorem imply

$$\max\{\mathrm{Vol}(P) : P \in \mathcal{P}^2(k)\} = \begin{cases} 9 & (k = 1), \\ 4(k+1) & (k \geq 2). \end{cases}$$

In case $d \geq 3$, $k \geqq 1$, the following conjecture is in [2]:

$$\max\{\mathrm{Vol}(P) : P \in \mathcal{P}^d(k)\} = (s_d - 1)^2(k+1),$$

where $\{s_i\}_{i \in \mathbb{N}}$ is the Sylvester sequence given by $s_1 = 2, s_i = s_1 s_2 \cdots s_{i-1} + 1$.

In case $d = 3$, $k = 1$, 2, the conjecture are solved ($cf.$ [2]).

In case of simplices, for $d \geq 4$, $k = 1$, G. Averkov, J. Krümpelmann, B. Nill [1] solved the conjecture: $\max\{\mathrm{Vol}(S) : S \in \mathcal{S}^d(1)\} = 2(s_d - 1)^2$.

In case $d = 3$, $k \geq 3$, even simplices' case has not resolved. We restrict the assumption of conjecture to lattice corner $3-$*simplices* and get the following result.

Theorem 1. *Let $k \geq 1$ be a fixed integer. Then the maximum normalized volume of lattice $3-$simplices in $\mathcal{CS}^3(k)$ is as follows:*

$$\max\{\mathrm{Vol}(\Delta) : \Delta \in \mathcal{CS}^3(k)\} = 36(k+1).$$

References

[1] Averkov, G., Krümpelmann, J., Nill, B., Largest integral simplices with one interior integral point: Solution of Hensley's conjecture and related results. Adv. Math. 274, 118–166 (2015)

[2] G. Balletti and A. M.Kasprzyk, Three-dimensional Lattice Polytopes with Two Interior Lattice Points, arXiv:1612.08918 math.CO, 2016.

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Statistical Mechanics Approach for Random Single Vertex Origami

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Abstract

We investigate the statistical property of a genrerative model of random single vertex origami to obtain understanding for the phase transition phenomena of the flat-foldability of single vertex origami to approach the average-case complexity of the flat-foldability problem. Average behavior on the ensemble of instances of single vertex origami diagram under the geometric property of them is controlled. A sudden change of the average number of cluster spins resulting from the contraction process is observed in the vicinity of a certain value of a geometric parameter of the diagrams.

1 Statistical Mechanics Model

The origami diagram which satisfies the necessary condition for flat-foldability required by Kawasaki's theorem, such as those in Fig.1(a), has a pre-folded diagram which shows positional-relationship of the facets after they are folded, as shown in Fig.1 (b). The vertical relationship or the (local) layer ordering of two facets j and k as shown in Fig.1(c), wheather one facet of the pair is below- or above- side of the another, can be represented by the binary variable $s_{i,k} \in \{1, -1\}$. For example, in the case of the figure in Fig.1(c) $s_{j,k}$ is defined to take its value $+1$ when the facet k is above the facet j in z-axis (the direction of height).

By representing the problem with the combination of the local layer-ordering, the flat-foldability problem of the origami becomes the combinatorial problem. To avoid some infeasible layer-ordering that are caused by an interpenetration of facets or so, some constraint terms which consists of the product of two (or four) spin variables. The binary combination of spin variabless that satisfy all constraint terms will be corresponding to the realizable flat-folding of the origami-sheet. The set of such constraint terms is the function to be optimized, which is called the energy function, or Hamiltonian, in the context of physics.

The constraints on the layer-ordering are classified into the following three types.

1. An intrusion of a facet into a crease which connects other two facets.
2. Some ordering among four facets connected to each sides of the two creases which are in geometrically coincidental position the pre-folded diagram.
3. A cyclic ordering of three facets which have areas shared with all each other in the pre-folded diagram.

The constraint type 1 appeares in the cases such as the facet k is sandwitched between i and j. The term which represents this constraint is described as

$$E^{(i)}_{ij,k} = \frac{1}{2}(1 - J_{(ik)(kj)} s_{ik} s_{kj}), \tag{1}$$

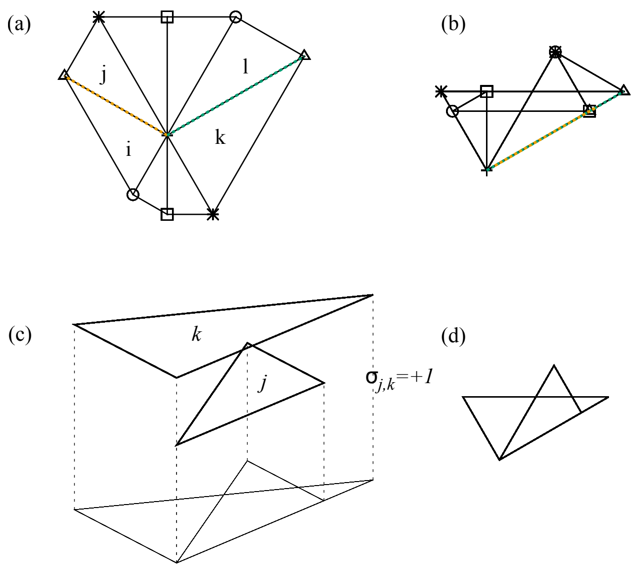

Figure 1: (a)Example of origami diagram. Each edge in the figure represents a crease. In this figure there are no overlaps of facets. (b)Corresponding pre-folded diagram, which describes the overlaps of facets when the figure (a) is folded along the creases. Each vertiex indicated by the same mark is the same as that in Fig. (a). (c)Schematic picture of introduction of the Ising variable to a local layer-ordering.

where $J_{(ik)(kj)}$ is the binary coefficient which is given depending on the index-labeling of facets.

The constraint type 2 is considered in cases that there are two creases partially coincide with each other and pairs of facets, called i, j and k, l, each connected by the creases. The example is exhibited in Fig.2. To give such permission and prohibition of layer-orderings, we need the product of four spin variables as

$$E^{(q)}_{ijkl} \;\;=\;\; \frac{1}{2}\Big(1 - K_{ijkl}s_{ik}s_{il}s_{jk}s_{jl}\Big), \tag{2}$$

where $K_{ijkl} \in \{1, -1\}$ is each given dependently on the index-labeling.

For the constraint type 3, at first the cyclic ordering of three facets i, j, and k is described with spin variables $\{s_{ij}, s_{ik}, s_{ik}\}$ as the combinations of variables such that one of the previous spin variables have the opposite sign from the other two. To prohibit these combinations, the set of terms of spins are introduced as

$$E^{(c)}_{ijk} \;\;=\;\; \frac{1}{4}(1 - L_{(ij)(jk)}s_{ij}s_{jk} - L_{(jk)(ki)}s_{jk}s_{ki} - L_{(ki)(ij)}s_{ki}s_{ij}), \tag{3}$$

where three coefficients $L_{(ij)(jk)}$ are given from the detail of the index-labeling.

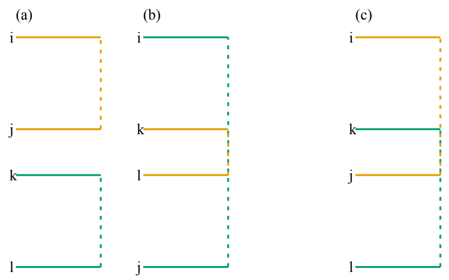

Figure 2: (Color online) (a),(b)Layer orderings of facets which are accepted under the coincident of two creases. (c) Layer ordering which is NOT accepted under the coincident of two creases. One connected pair of facets must penetrate the another connected part of the remanent two facets.

2 Single-vertex origami diagram

Various single-vertex origami diagrams such as exhibited in Fig.1(a) are randomly generated. We asymptotically evaluate the behavior in the limit of an infinite number of facets $n \to \infty$ from the sequences of those with finite number of facets $n = 12, 24, 48....$ When n increases as such, the number of spin variables, N, are coincidently increases as $N = 66, 276, 1128....$

We generated the instances so that the angle of each facet around the central vertex is an integer multiple of a discrete unit and the value of the alternating sum around the center is zero. The purpose of introducing a minimum unit to the angle of the facets is to make the diagram easy to have the coincidence of the two creases. By making the minimum unit value finer, each facet can take more diverse values, and the total number of coincidences is to decrease. Conversely, if the value of the minimum unit is coarsened, the total number of coincidences will increase. We can control the total number of coinsidences by changing the value of the minimum unit.

We apply the contraction procedure introduced in the next chapter to try to reduce the number of variables involved in the combinatorial optimization of search for flat-foldings. As the contraction process is proceded with a polynomial time of the system size N, the amount of computational effort is thought to be concentrated to the resulting combinatorial optimization problem consisting of cluster spins. Tnus the number of cluster spins is thought to be related to the essential hardness of the problem. The average number of cluster spins contained in each instance varies systematically dependent on the number of coincidences of two creases. A sudden change of the average number of cluster spins is observed in the vicinity of a certain value of the number of coincidences.

3 Appendix : contraction of spin variables

When a two-body interaction term is given between two spin variables, the combination of spin variables that satisfies the constraint is uniquely determined except for total inversion. Therefore, we translate the constraint that prohibits intrusion into an allowed relationship between the spin variables. For example, the energy function for the fold diagram in Fig.3 originally has 41 terms of $E^{(i)}_{ij,k}$-type, in addition to a term of $E^{(q)}_{ijkl}$-type and 84 terms of

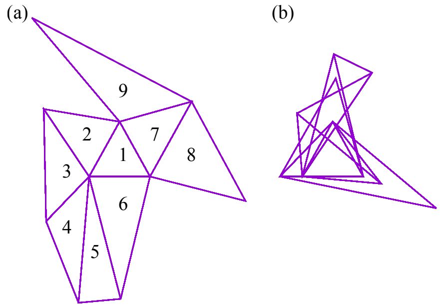

Figure 3: (Color online) (a)Example of origami diagram. Each edge in the figure represents a crease. In this figure there are no overlaps of facets. (b)Corresponding pre-folded diagram, which describes the overlaps of facets when the figure (a) is folded along the creases.

Table 1: Correspondence among cluster spins and sets of elemental spins.

Cluster spin	Set of elemental spins $s_{i,j}$ included
$\mathscr{C}_1 = 1$	$\{s_{1,2}, -s_{1,3}, -s_{1,4}, -s_{1,5},$ $-s_{1,6}, -s_{1,7}, -s_{1,8}, s_{1,9},$ $-s_{2,3}, -s_{2,4}, -s_{2,5}, -s_{2,6},$ $-s_{2,7}, -s_{2,8}, s_{2,9}, s_{3,4},$ $s_{3,5}, s_{3,6}, s_{3,7}, s_{3,8},$ $s_{3,9}, s_{4,7}, s_{4,8}, s_{4,9},$ $s_{5,7}, s_{5,8}, s_{5,9}, s_{6,7},$ $s_{6,8}, s_{6,9}, -s_{7,8}, s_{7,9}, s_{8,9}\}$
$\mathscr{C}_2 = 1$	$\{s_{4,5}, s_{4,6}, s_{5,6}\}$

$E^{(c)}_{ijk}$-type. In these 41 terms, a term has the form that is $\frac{1+s_{1,2}s_{2,6}}{2}$. This term is trnslated into the relationship that is $s_{1,2} = -s_{2,6}$. Thus, the energy function of the fold diagram in Fig.3 is redescribed into a form consisting of 2 spin variables $\{\mathscr{C}_1, \cdots, \mathscr{C}_{11}\}$. See Table1 for the relationship between variables $\{\mathscr{C}_1, \cdots, \mathscr{C}_{11}\}$ and variables $\{s_{1,2}, \cdots, s_{9,10}\}$. The detail of this contraction procedure is presented at the international conference *The 24th Conference of the Japan Conference on Discrete and Computational Geometry, Graphs, and Games (JCDCG3 2022)*[1].

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Continuous Folding of the Surface of a Regular Simplex onto its Facet

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We provide a new method for folding the surface of a 4-dimensional regular simplex onto its facet continuously that requires one sixth of the entire surface for moving creases. Whether the surface of a polyhedron of a flexible material such as paper can be flattened without cutting or stretching is a problem that has been investigated. (See [2], p.279). This problem was solved for all convex polyhedra in [1, 5] using moving creases to change the shapes of some faces, which follows from Cauchy's rigidity theorem.

We proved that any regular simplex (in general, an n-dimensional convex polytope) can be continously folded in any $(n-1)$-dimensional facet in a joint paper [1]. However, the portions of the moving creases occupy almost the entire surface except at most two facets. For example, for a tetrahedron using the method shown in [3, 6], the portions of the moving creases occupy one twelfth of the entire surface, and the method in [1] results in three fourth of the portions being occupied (see Fig. 1).

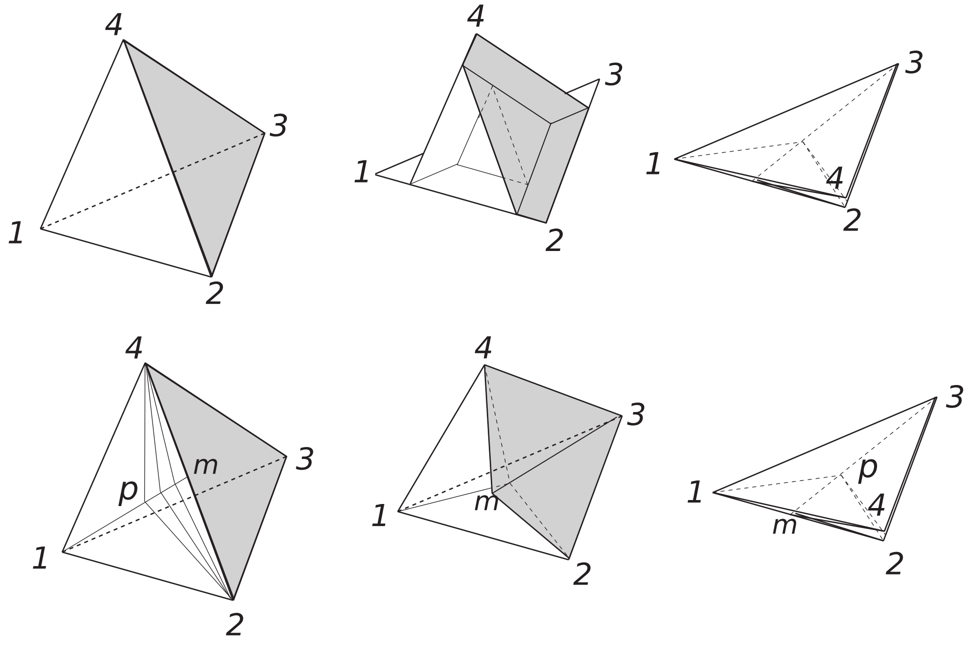

Figure 1: Continuous flattening of the surface of a regular tetrahedron using two methods, where the upper figures are for the method in [1] and the lower figures for the method in [3, 6].

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Note that we proved in [4] that there is a continuous folding of the 2-dimensional skeleton, the set of triangular faces, onto its facet. This process involves empty facet interiors. Therefore, the situation differs from the theorem in this paper because the facets are considered 3-dimensional bodies.

Theorem 1. *The surface of a 4-dimensional regular simplex can be continuously folded onto any of its facets such that the total volume used for the moving creases is one sixth of the surface volume.*

For points $p_1, p_2, ..., p_n$ with $n \geq 2$ in 4-space we denote $< p_1p_2...p_n >$ its covexhull and $(p_1p_2...p_n)$ the center (of gravity). So (p_1p_2) means the midpoint of p_1 and p_2. Let P be the 4-dimensional regular simplex with 5 vertices $\{v_i : i = 0, 1, 2, 3, 4\}$ in 4-space, which are denoted in short $0, 1, 2, 3, 4$, respectively, and whose edge length is l. The outline of the motions is as follows.

Motion of the vertices. The facet $< 1234 >$ is fixed and the facet $< 0234 >$ is moved onto $< 1234 >$ by rotating about the face $< 234 >$. Hence the vertex 0 is moved on the vertex 1 along the circular arc in the circle obtained as the intersection of three 3-spheres of the radius l with center $1, 2$ and 3.

Motion of the edge $< 01 >$. The edge is folded in halves at the midpoint $m = (01)$ which is moved onto the midpoint (12) along the circular arc in the intersection of two spheres of the radius $(\sqrt{3}/2)l$ with center $2, 3$ and the spehre of the radius $l/2$ with center 1 such that for each moment m stays in the hyperplane bisecting two points 1 and 0.

Motion of the three faces attaching to the edge $< 01 >$. Two faces $< 013 >$ and $< 014 >$ are in halves folded onto $< 1(12)3 >$ and $< 1(12)4 >$, respectively. The face $< 012 >$ is folded onto $< 12(123) >$ with moving creases.

Then, moving creases occupy one third of the facet $< 0124 >$ which is the pyramid $< 01(012)4 >$, and one half of the facet $< 0123 >$ which is the union of the three pyramids $< 01(012)3 >, < 12(012)(0123) >$, and $< 02(012)(0123) >$.

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Numbers of Weights of Convex Quadrilaterals in Weighted Point Sets

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Abstract

Let P be a set of points in the plane. P is said to be *in general position* if no three of its elements are collinear. All point sets considered in this talk are in general position in the plane. We say P *contains* a convex k-gon if P contains k elements that are vertices of a convex k-gon.

Erdős and Szekeres [2] proved that for any integer $k \geq 3$, there is an integer $N(k)$ such that any set of at least $N(k)$ points contains a convex k-gon. In 1984, Erdős [1] asked the minimum number $\mathrm{conv}_k(n)$ of convex k-gons contained in a point set with n elements. In particular, for $k = 4$, this problem is equivalent to the problem of determining the *rectilinear crossing number* of K_n, and has been studied extensively for a long time.

P is called a *weighted point set* if each point is assigned a number called a *weight*. We denote by $\mathcal{P}(n)$ the collection of weighted point sets P with n elements each of which receives a different weight in $\{1, 2, \dots, n\}$. For a polygon Q with vertices in $P \in \mathcal{P}(n)$, we denote by $w(Q)$ the sum of the weights of its vertices. Let $f(P)$ denote the total number of different weights of convex quadrilaterals contained in $P \in \mathcal{P}(n)$, and let $F(n) = \min_{P \in \mathcal{P}(n)} f(P)$. It is shown in [3] that

$$f_4(6) = 1 \text{ and } f(n) \leq 2n - 9 \text{ for } n \geq 7.$$

A lower bound is also shown in [3, Theorem2]:

$$n - 5 \leq F(n) \quad \text{for } n \geq 6.$$

In this talk, we show the following lower bound:

Theorem 1. $\dfrac{4n-21}{3} \leq F(n)$ *for* $n \geq 6$.

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Contributed Talks - Computational Geometry

When Can You Tile an Integer Rectangle with Integer Squares?

MIT CompGeom Group[*], Zachary Abel[†], Hugo A. Akitaya[‡], Erik D. Demaine[†], Adam C. Hesterberg[§], Jayson Lynch[†]

Abstract

In this paper, we characterize which rectangles of integer side lengths can be tiled by squares each of integer side length at least 2, as follows:

(I) $2 \times n$ and $4 \times n$ rectangles can tiled exactly when n is even.

(II) $3 \times n$ rectangles can be tiled exactly when $n \equiv 0 \pmod 3$.

(III) $m \times n$ rectangles for all $m \geq 5, n \geq 20$ and $m \geq 20, n \geq 5$ can be tiled.

(IV) Table 1 specifies tileability for all remaining m, n (indeed, for all $m, n < 20$).

In particular, Table 1 indicates successful tilings for all $10 \leq m, n \leq 20$, so combined with (III), we obtain that the $m \times n$ rectangle is tileable without 1×1 squares for all $m, n \geq 10$. Our tilings use only 2×2, 3×3, 5×5, and 7×7 squares, so our result can also be cast in terms of restricting the set of allowed square sizes to these four. See [1, 2] for related work on tiling rectangles with a few square sizes.

$m \backslash n$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	✓		✓		✓		✓		✓		✓		✓		✓		✓	
3		✓			✓			✓			✓			✓			✓	
4	✓		✓		✓		✓		✓		✓		✓		✓		✓	
5				✓	✓				✓	✓	✓			✓	✓	✓	✓	
6	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
7					✓	✓			✓		✓	✓	✓		✓	✓	✓	✓
8	✓		✓		✓		✓		✓		✓		✓	✓	✓	✓	✓	✓
9		✓			✓			✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
10	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
11				✓	✓				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
12	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
13					✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
14	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
15		✓		✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
16	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
17				✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
18	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
19					✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Table 1: Which integer $m \times n$ rectangles, for $2 \leq m, n \leq 19$, admit tilings with squares of side length at least 2. ✓ indicates when a tiling was found by brute force. Code available at https://github.com/MIT-CompGeom/tiling-rectangles-with-squares

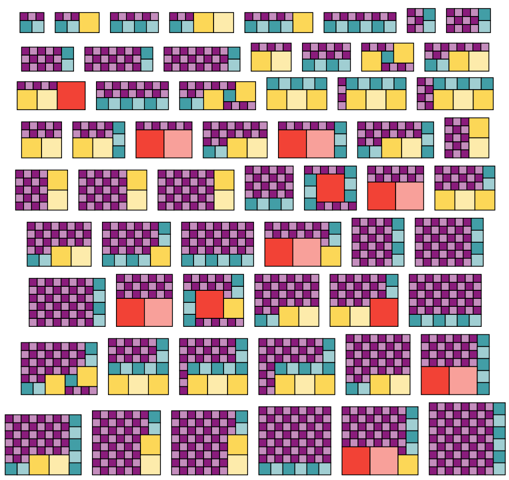

Table 2: Tilings found by brute force corresponding to ✓s in Table 1 except for dimensions with a common factor. 2×2, 3×3, 5×5, and 7×7 squares are purple, teal, yellow, and red, respectively.

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On the Eigen Values of Laplace Operator Defined on the Dodecahedron Metric Graph

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Abstract

Of concern is the eigenvalue problem of the Laplace operator on the Dodecahedron metric graph. The study is part of a more general problem of the eigenvalues of the Laplace operator on the Platonic Solids metric graphs. A compact metric graph is a graph, in which the edges are identified by finite line segments, enabling one-dimensional calculus to be done on this structure. Certain conditions must be imposed on vertices, which is analogous to boundary conditions in differential equations. In this study, the Neumann-Kirchoff condition, along with compatibility conditions are imposed on the metric graph. The explicit computation of the eigenvalues is carried out on a mobile computer using Wolfram Mathematica. Our results match those of Lipovsky and Exner (2019), in which heavy operator theoretic tools were used.

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On the stretch factor of Delaunay triangulations of points in convex position

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(This talk is based on joint work with Rong Chen, Dalian Maritime University, China.)

Abstract

Let S be a set of n points in the plane, and let $DT(S)$ be the planar graph of the Delaunay triangulation of S. For a pair of points $a, b \in S$, denote by $|ab|$ the Euclidean distance between a and b. Denote by $DT(a, b)$ the shortest path in $DT(S)$ between a and b, and let $|DT(a, b)|$ be the total length of $DT(a, b)$. $DT(S)$ can be used to approximate the complete graph of S in the sense that the stretch factor $\frac{|DT(a,b)|}{|ab|}$ is upper bounded by a constant, independent of S and n. The currently known best factor for a set of planar points is 1.998.

In this wor, we prove that for a set S of points in convex position (i.e., they form the vertices of a convex polygon), the stretch factor of $DT(S)$ is 1.74. It not only improves upon the previously known factor 1.84 for points in convex position, but also shows a large possibility of obtaining the same stretch factor for points in general position.

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Contributed Talks - Graph Theory

k-Ramsey Numbers of Stars

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Abstract

For an integer $k \geq 2$, a balanced complete k-partite graph of order $m \geq k$ is the complete k-partite graph in which every partite set has vertices $\lfloor m/k \rfloor$ or $\lceil m/k \rceil$. For bipartite graphs $G_1, G_2, \ldots, G_\ell$ and an integer k with $2 \leq k \leq R(G_1, G_2, \ldots, G_\ell)$, define the k-Ramsey number $R_k(G_1, G_2, \ldots, G_\ell)$ as the smallest positive integer n_0 such that every ℓ-coloring of a balanced complete k-partite graph of order n_0 produces monochromatic subgraph G_i in color i for some $i \in \{1, 2, \ldots, \ell\}$. This definition is the generalization of k-Ramsey number introduced by Andrews et al. in 2017. They presented a formula for the k-Ramsey number $R_k(K_{1,s}, K_{1,t})$ of every two stars $K_{1,s}$ and $K_{1,t}$ $(s, t \geq 2)$ and every integer k with $2 \leq k \leq R(K_{1,s}, K_{1,t})$.

In this paper, we determine the exact values for $R_k(K_{1,n_1}, \ldots, K_{1,n_\ell})$ where $k, \ell, n_1, \ldots, n_\ell$ are integers with $2 \leq k < R(K_{1,n_1}, \ldots, K_{1,n_\ell})$, $\ell \geq 2$, $N = \sum_{i=1}^{\ell} n_i$ and $n_1, \ldots, n_\ell \geq 3$ provided that $N - \ell + \left\lfloor \frac{N-\ell}{k-1} \right\rfloor$ is even. We also give an upper bound and a lower bound for the case that $N - \ell + \left\lfloor \frac{N-\ell}{k-1} \right\rfloor$ is odd where the gap is only 2.

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On Forbidden Subgraphs of $(C_m|S_n)$-Magic Graphs

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Abstract

Let H_1 and H_2 be two non-isomorphic graphs. A graph G admits an $(H_1|H_2)$-covering if G admits neither H_1-covering nor H_2-covering, but each edge of G belongs to a subgraph isomorphic to H_1 or H_2.

Let G admits an $(H_1|H_2)$-covering. A total labeling f of G is called an (H_1, H_2)-magic if there exist two positive integers k_1 and k_2 such that $w(H') = \sum_{v\in V(H')} f(v) + \sum_{e\in E(H')} f(e) = k_1$ for each subgraph H' of G isomorphic to H_1 and $w(H'') = \sum_{v\in V(H'')} f(v) + \sum_{e\in E(H'')} f(e) = k_2$ for each subgraph H'' of G isomorphic to H_2. In this case, G is said to be $(H_1|H_2)$-magic. Furthermore, f is called $(H_1|H_2)$-supermagic if $f(V(G)) = \{1, 2, \ldots, |V(G)|\}$.

This talk provides some forbidden subgraphs of $(C_m|S_n)$-magic graph for $3 \leq n < m$. For two connected graphs H_1 and H_2 and for two positive integers $m, n \geq 3$, we provide some necessary and sufficient conditions for $mH_1 \cup nH_2$ to be $(H_1|H_2)$-supermagic. Finally, we characterize $kC_m \cup lS_n$ that is $(C_m|S_n)$-supermagic for every $|k - l| \leq 1$.

The strong rainbow vertex-connection number of comb product of a path and a connected graph

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Abstract

A vertex-colored connected graph G is strongly rainbow vertex-connected if every two vertices of G are connected by a shortest path whose internal vertices have distinct colors. Such a path is called a rainbow geodesic. The strong rainbow vertex-connection number of G, denoted by $\mathrm{srvc}(G)$, is known as the minimum number of colors needed in order to make G strongly rainbow vertex-connected. In this paper, we estimate sharp lower and upper bounds of the strong rainbow vertex-connection number of comb product $P_n \rhd_o H$ and characterize connected graphs H so that the strong rainbow vertex-connection number of $P_n \rhd_o H$ attains the lower bound. We also determine the exact values of the strong rainbow vertex-connection number of $P_n \rhd_o H$ for some connected graphs H.

The multiset dimension of prisms

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Let $G = (V, E)$ be a simple connected graph and $W \subset V$. For $v \in V$, we define the **m-code** of v, denoted by $\mathrm{mc}_G(v)$ or simply $\mathrm{mc}(v)$, to be the multiset of distances between v and the vertices in W; that is $\mathrm{mc}(v) = \{d(v, w) \mid w \in W\}$. If every pair of distinct vertices in G have distinct m-codes, then W is called an **m-resolving set** of G. If G has an m-resolving set, then the cardinality of a smallest m-resolving set is called the **multiset dimension** of G, denoted by $\mathrm{md}(G)$. In this paper, we show that if $n \geq 17$, then the prism $C_n \,\square\, K_2$ has multiset dimension 3.

The multiset dimension was introduced by Saenpholphat [4] and Simanjuntak [5]. Chartrand, et al. [1] introduced an equivalent concept called the ID number. Some of the known results that are used in the study are enumerated below.

Proposition 1 ([1])**.** *If $G = (V, E)$ is a connected graph, W a subset of V, $w \in W$, and $v \in V - W$, then $mc(w) \neq mc(v)$.*

Proposition 2 ([1],[4],[5])**.** *No connected graph has multiset dimension 2.*

Theorem 3 ([1],[4],[5])**.** *A nontrivial connected graph G has $md(G) = 1$ if and only if G is a path.*

For the next result, let P_n be the path $(0, 1, 2, \ldots, n-1)$ of order $n \geq 4$. We define a **symmetric** subset W of $V(P_n)$ to be one with the property $i \in W$ if and only if $n - i \in W$, for each $i \in V(P_n)$.

Theorem 4 ([2])**.** *Let $n \geq 4$. If $W \subset V(P_n)$ contains 0 and $n-1$ and is not symmetric, then W is an m-resolving set of P_n.*

Results

Let $G = (V, E)$ be a simple connected graph and W a subset of V with cardinality k. For any $v \in V$, we let $\mathrm{mc}(v) = \{d_1(v), d_2(v), \ldots, d_k(v)\}$ where $d_i(v) \leq d_{i+1}(v)$, for $i = 1, 2, \ldots, k-1$. For convenience, we also define $\mathrm{sum}_2(v) = d_1(v) + d_2(v)$; that is, $\mathrm{sum}_2(v)$ is the sum of the two shortest distances from v to the vertices in W. Consequently, we have the following.

Observation 5. *Let $G = (V, E)$ be a simple connected graph and W a subset of V with cardinality 3. If $u, v \in V$ with $sum_2(u) \neq sum_2(v)$, then $mc(u) \neq mc(v)$.*

In the next theorem we construct an m-resolving set of minimum cardinality for C_n, $n \geq 9$, that is different from the set used in [1], [5], and [4] to show that $\text{md}(C_n) = 3$. It can be proved using Theorem 4, Proposition 1, and Observation 5.

Theorem 6. *Let $n \geq 9$, $G = C_n$ with $V(G) = \mathbb{Z}_n$ and $E(G) = \{\{i, i+1\} \mid 0 \leq i \leq n-1\}$, with addition done modulo n. Then $W = \{0, \lfloor n/4 \rfloor, 2\lfloor n/4 \rfloor + 1\}$ is an m-resolving set of G.*

In [3], Kono and Zhang have established that prisms $C_n \square K_2$ have an m-resolving set if and only if $n \geq 6$. However, finding the minimum cardinality of such m-resolving sets has not yet been addressed.

For the remaining discussion, we let G be the prism $C_n \square K_2$ where $n \geq 17$. We let

$$V(G) = \{(i,j) \mid 0 \leq i \leq 1, 0 \leq j \leq n-1\}, \text{ and}$$
$$E(G) = \{\{(i,j),(i,j+1)\} \mid 0 \leq i \leq 1, 0 \leq j \leq n-1\} \cup \{\{(0,j),(1,j)\} \mid 0 \leq j \leq n-1\}.$$

For $i = 0$ or 1, we denote the cycle $((i,0),(i,1),\ldots,(i,n-1),(i,0))$ by $C_n(i)$.

Suppose $W \subset V(C_n(0))$ with k elements. Then, for any j, $0 \leq j \leq n-1$, let u and v be the vertices $(0,j)$ and $(1,j)$, respectively. Then $d_i(v) = d_i(u) + 1$ for any i, $1 \leq i \leq k$, and $\text{sum}_2(v) = 2 + \text{sum}_2(u)$. Hence, we can make the following observation.

Observation 7. *Let $G = C_n \square K_2$ and $W \subset V(C_n(0))$ an m-resolving set of $C_n(0)$. Then the m-codes of the vertices of $C_n(1)$ are distinct.*

Here is our main result.

Theorem 8. *Let $G = C_n \square K_2$. Then $W = \{(0,0),(0,\lfloor n/4 \rfloor),(0,2\lfloor n/4 \rfloor + 1)\}$ is an m-resolving set of G. Hence, $md(G) = 3$.*

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Locating rainbow connection numbers of the edge corona of trees with complete graphs

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Abstract

Let $G = (V(G), E(G))$ be a simple, connected, and finite graph. For any $k \in \mathbb{N}$, a rainbow vertex k-coloring of G is a function $c : V(G) \longrightarrow \{1, 2, ..., k\}$ such that for every two distinct vertices u and v in $V(G)$ there exists a $u - v$ path whose internal vertices have distinct colors. Such path is called a rainbow vertex path. The rainbow vertex connection number of G, denoted by $rvc(G)$, is the smallest positive integer k so that G has a rainbow vertex k-coloring. For $i \in \{1, 2, ..., k\}$, let R_i be the set of vertices with color i and $\Pi = \{R_i, R_2, ..., R_k\}$ be an ordered partition of $V(G)$. The rainbow code of a vertex v of $V(G)$ with respect to Π is defined as the k-tuple $rc_\Pi(v) = (d(v, R_1), d(v, R_2), ..., d(v, R_k))$, where $d(v, R_i) = \min\{d(v, y)|y \in R_i\}$ for each $i \in \{1, 2, ..., k\}$. If every vertex of G has distinct rainbow codes, then c is called a locating rainbow k-coloring of G. The locating rainbow connection number of G, denoted by the $rvcl(G)$, is defined as the smallest positive integer k such that G has a locating rainbow k-coloring.

Let G and H be two graphs on disjoint sets of $|V(G)|$ and $|V(H)|$ vertices, $|E(G)|$ and $|E(G)|$ edges, respectively. The edge corona of G and H denoted by $G \diamond H$ is defined as the graph obtained by taking one copy of G and $E(G)$ copies of H, and then joining two end-vertices of the j-th edge of G to every vertex in the j-th copy of G, for $j \in \{1, 2, ..., |E(G)|\}$. In this paper, we determine the upper and lower bounds of the locating rainbow connection number for the class of graphs resulting from the edge corona of a tree with a complete graph. Furthermore, we demonstrate that these upper and lower bounds are tight.

Saturated Partial Embeddings of Planar Graphs

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(This talk is based on joint work with Nika Salia.)

Abstract

In this work, we study how far one can deviate from optimal behavior when drawing a planar graph on a plane. In particular, we define the plane saturation number, $\mathrm{ps}(G)$, of a planar graph G as the minimum number of edges in a subgraph $H \subseteq G$ such that there exists a planar embedding of H where adding any edge (possibly with a new vertex) to the embedding would either violate planarity or make the resulting graph no longer a subgraph of G.

We investigate how small $\mathrm{ps}(G)$ can be relative to the number of edges, $|E(G)|$, in G. While there exist planar graphs where $\mathrm{ps}(G)/|E(G)|$ is arbitrarily close to 0, we show that for all twin-free planar graphs, $\mathrm{ps}(G)/|E(G)| > 1/16$, and that there exist twin-free planar graphs where $\mathrm{ps}(G)/|E(G)|$ is arbitrarily close to 1/16.

A GENERAL METHOD TO FIND THE SPECTRUM AND EIGENSPACES OF THE k-TOKEN OF A CYCLE

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(This talk is based on joint work with Miquel Àngel Fiol, Arnau Messegué, and Mónica A. Reyes.)

Abstract

The k-token graph $F_k(G)$ of a graph G is the graph whose vertices are the k-subsets of vertices from G, two of which being adjacent whenever their symmetric difference is a pair of adjacent vertices in G.

In this talk, we are going to present a general method to find the spectrum and eigenspaces of the k-token graph $F_k(C_n)$ of a cycle C_n. This method is based on the theory of lift graphs and the recently introduced theory of over-lifts, which are going to explain.

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On the Metric Dimension and Spectrum of Graphs

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(This talk is based on joint work with Edy Tri Baskoro.)

Abstract

The problem of determining the metric dimension of a graph is well-known to be NP-complete. Thus, there is no efficient algorithm to solve this problem in general. On the other hand, as computation developments have grown rapidly for the last century, the use of matrices to represent a graph has given many useful facts to determine the structure of the graph, e.g., by using their spectrum. Moreover, the algorithms for finding the eigenvalues of a matrix have been developed such that their computation is relatively quick. Therefore, we pose the following question: is there any connection between the metric dimension of a graph and its spectrum? In this talk, we present some positive results on this question.

Rainbow Connection Numbers of An s-Overlapping r-Uniform Homogeneous Caterpillar Hypergraph

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Abstract

In 2014, Carpentier et al. introduced the concept of the rainbow connection in hypergraphs. This concept is an extension of the rainbow connection in a graph introduced by Chartrand et al. in 2008. This concept has been implemented on an r-uniform minimally connected hypergraph, an r-uniform cycle hypergraph, and an r-uniform complete multipartite hypergraph. In our research, we apply this concept to s-overlapping r-uniform hypergraphs with size t. Let $r \geq 2$, $1 \leq s < r$, and $t \geq 1$ be integers. An s-overlapping r-uniform hypergraph with size t, denoted by $\mathcal{H}^r_{s,t}$, is an r-uniform connected hypergraph where s is the maximum cardinality of the vertex set resulting from the intersection of each pair of edges in the hypergraph. In our initial result, we determined the rainbow connection number of an s-overlapping r-uniform interval hypergraph with size t, denoted by $\mathcal{P}^r_{s,t}$. The hypergraph $\mathcal{P}^r_{s,t} = (X(\mathcal{P}^r_{s,t}), \mathcal{E}(\mathcal{P}^r_{s,t}))$ has the vertex set $X(\mathcal{P}^r_{s,t}) = \{v_1, v_2, \ldots, v_{(t-1)(r-s)+r}\}$ and the edge set $\mathcal{E}(\mathcal{P}^r_{s,t}) = \{E_1, E_2, \ldots, E_t\}$ where

$$E_i = \{v_{(i-1)(r-s)+1}, v_{(i-1)(r-s)+2}, \ldots, v_{(i-1)(r-s)+r}\} \text{ for all } i \in \{1, 2, \ldots, t\}.$$

In this talk, we determine the rainbow connection number of an s-overlapping r-uniform homogeneous caterpillar hypergraph.

Local Inclusive Distance Antimagic Coloring of Graphs

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Abstract

For a graph G, a bijection $f : V(G) \to [1, |V(G)|]$ is a *local inclusive distance antimagic (LIDA) labeling* of G if $w(u) \neq w(v)$ for every two adjacent vertices $u, v \in V(G)$ with $w(u) = f(u) + \sum_{x \in N(u)} f(x)$, or equivalently, $w(u) = \sum_{x \in N[u]} f(x)$. A graph is said to be local inclusive distance antimagic (LIDA) if it admits a LIDA labeling. Observe that w is a proper coloring of G. Thus, if f is a LIDA labeling of G, then the function w is referred as the *coloring of G induced by f*, and the number $w(u)$ is referred as the *color* (or weight) of the vertex $u \in V(G)$. The *local inclusive distance antimagic chromatic number* of G, denoted by $\chi_{lida}(G)$, is the minimum number of colors taken over all colorings induced by LIDA labelings of G. We determine $\chi_{lida}(G)$ for some classes of graphs G. In this paper, we investigate the LIDA chromatic number of several classes of graphs and some graph operations.

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Locating Chromatic Number for Corona Operation of Path and Cycle

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Abstract

The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [1]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph. Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \to \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The corona operation of P_n and C_m, denoted by $P_n \odot C_m$) is defined as the graph obtained by taking one copy of P_n and $|V(P_n)|$ copies of C_m and then joining all the vertices of the i^{th} -copy of C_m with the i^{th}-vertex of P_n [2]. In this paper, we will discuss the locating chromatic number for the corona operation of path and cycle. The locating chromatic number of $P_n \odot C_3$ is 5 for $3 \leq n < 7$ and 6 for $n \geq 7$. Next, $\chi_L(P_n \odot C_4)$ is 5 for $3 \leq n < 6$ and 6 for $n \geq 6$.

Keywords : Locating chromatic number, Corona operation.

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The Maximum Girth of Rainbow Cycles in Strong Edges Colored Graphs

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Abstract

A graph with an edge coloring is good PRCF (*proper rainbow-cycle-forbidding*) if it uses the proper edge-coloring and has absolutely no rainbow cycles. If these criteria are not fulfilled, then the graph is called bad PRCF. Proper edge coloring defines the boundary that each adjacent vertices cannot have a same color. With this coloring, it is quite difficult to find a graph that contains rainbow cycles with girth greater than five. In this research, edge coloring with stricter rules was used, namely strong edge coloring. Strong edge coloring has a rule that every two adjacent edges and two edges with the same neighbor, cannot get the same color. This coloring makes it possible to obtain a rainbow cycle with a larger girth. That way, many graphs can be included in the scope of this research, especially graphs that contain cycles with a girth of more than 5, as well as each of its characteristics. Using strong edge coloring also termed in this research that a graph with strong edge coloring and does not contain rainbow cycles as good SRCF (*strong rainbow-cycle-forbidding*) and if one or both things are not fulfilled then it is called bad SRCF. This research also introduces joint k-distance graphs between two labeled graphs, namely graphs obtained by connecting the i-th vertex in the first graph to vertices of distance k from the i-th vertex in the second graph. Joint k-distance is used because it is possible to form a cycle on the constructed graph. For joint k-distance of some special graph, the maximum girth value of the cycle is obtained. The maximum value of girth of the rainbow cycle is also obtained.

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On the locating rainbow connection number of the edge-comb product of a path or a complete graph with a complete graph

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Abstract

This paper limits graphs to simple, finite, and connected graphs. Let ℓ be an integer, G and H be graphs, $V(G)$ be the set of vertices of graph G, and $E(G)$ be the set of edges of graph G. A rainbow vertex ℓ-coloring of G is a function $f{:}V(G){\to}\{1, 2, \ldots, \ell\}$ such that for every two distinct vertices in G, there exists a rainbow vertex path that connects the vertices. A rainbow vertex path is a path whose internal vertices have distinct colors. For a vertex $v \in V(G)$ and a subset $R \subseteq V(G)$, the distance between v and R is $\mathrm{d}(v, R) = \min\{\mathrm{d}(v, x) | x \in R\}$. For $i \in \{1, 2, \ldots, \ell\}$, let R_i be the set of vertices with color i. Form an ordered partition $\Pi = \{R_1, R_2, \ldots, R_\ell\}$ of $V(G)$. The rainbow code of a vertex v with respect to Π is defined as the ℓ-tuple

$$\mathrm{rc}_\Pi(v) = (\mathrm{d}(v, R_1), \mathrm{d}(v, R_2), \ldots, \mathrm{d}(v, R_\ell)).$$

If $\mathrm{rc}_\Pi(v) \neq \mathrm{rc}_\Pi(w)$ for every two distinct vertices $v, w \in V(G)$, then f is called a locating rainbow ℓ-coloring of G. The smallest positive integer ℓ such that G has a locating rainbow ℓ-coloring is called the locating rainbow connection number of G, denoted by $\mathrm{rvcl}(G)$.

An orientation of an undirected graph G is an assignment of precisely one direction to each of the edges of G. Let O be an orientation of G and $\vec{e}$ be an oriented edge of H. The edge-comb product of G (under the orientation O) and H on $\vec{e}$, denoted by $G^o \rhd_{\vec{e}} H$, is a graph obtained by taking one copy of G and $|E(G)|$ copies of H and identifying the j-th copy of H at the edge $\vec{e}$ to the j-th edge of G, where the two edges have the same orientation.

In this paper, we determine the locating rainbow connection number of the edge-comb product of a path with a complete graph and a complete graph with a complete graph.

Strong and Local Strong Rainbow Connection of Corona Product Graphs Involving Wheel Graphs

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(This talk is based on joint work with Denny Riama Silaban, Qonita Wafa Salsabila, Muhamad Alchem Nuravian Permana.)

Abstract

Let c be the edge coloring of a graph G. A rainbow u-v geodesic in G is a shortest path between u and v with all edges has different color. Graph G is said to be strongly rainbow connected if every pair of vertices in G connected by a rainbow geodesic. The strong rainbow connection number of G, $src(G)$, is the minimum number of colors needed to make G strongly rainbow connected. Let d be a positive integer, d-local strong rainbow coloring such that every pair of vertices of distance up to d connected by rainbow geodesic. A d-local strong rainbow connection number of G, $lsrc_d(G)$, as the minimum number of colors needed to make G being d-local strong rainbow connected. In this talk, we give the strong rainbow connection number and d-local strong rainbow connection number of corona product of graph that involving a wheel graph.

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Determination of all graphs whose eccentric graphs are clusters

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Abstract

A disconnected graph G is called a cluster if each component of G is a complete graph with order at least two. That is, $G = \bigcup_{i=1}^{n} K_{p_i}(n \geqq 2, p_i \geqq 2)$. J. Akiyama, K. Ando and D. Avis showed in Lemma 2.1 of [1] that G is equi-eccentric if the eccentric graph G_e is a cluster. In this paper we determined all graphs whose eccentric graphs are clusters, which is an extension of Lemma 2.1 in [1]. We also mention a few applications of eccentric graphs.

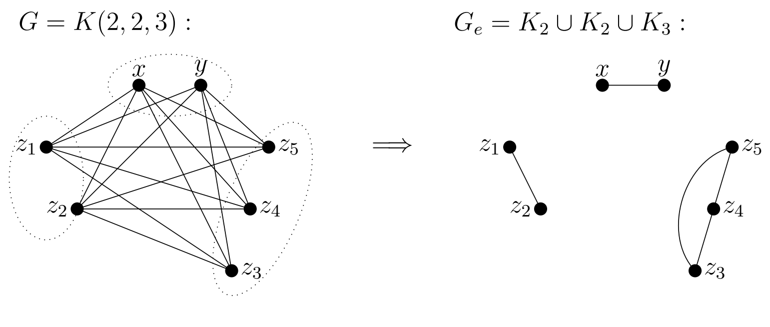
Let $G = (V(G), E(G))$ be a simple undirected graph. A disconnected graph G is called a **cluster** if it is union of complete graphs $\bigcup_{i=1}^{n} K_{p_i}(n \geqq 2, p_i \geqq 2)$.
The **eccentricity** $e(v)$ of a vertex v in $V(G)$ is defined by $e(v) = \max_{u \in V(G)} d(u, v)$, where $d(u, v)$ stands for the length of the shortest path in G between u and v. We denote by $G_e = (V(G_e), E(G_e))$ the **eccentric graph** based on G, where the vertex set $V(G_e)$ is identical to $V(G)$ and $uv \in E(G_e) \Leftrightarrow d(u, v) = \min(e(u), e(v))$. We denote $r(G)$ the **radius** of a graph G, which is defined as $r(G) = \min_{v \in V(G)} e(v)$. The complement of a graph G is denoted by $\overline{G}$.

The following theorems are our main results.

Theorem 1. *If $r(G) \geqq 2$, then $(G + \overline{K_n})_e = \overline{(G + \overline{K_n})} = \overline{G} \cup K_n, n \geqq 2$.*

Theorem 2. *A graph whose eccentric graph is a cluster, i.e., $G_e = \cup_{i=1}^{n} K_{p_i}(n \geqq 2, p_i \geqq 2$ for any $i(1 \leqq i \leqq p), \Sigma_{i=1}^{n} p_i = p)$, if and only if G is either a cycle C_{2p} or a complete n-partite graph $K(p_1, p_2, \ldots, p_n)$.*

The following figure shows an example of G_e obtained from $G = K(2, 2, 3)$.



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On the Distinguishing Chromatic Number of Disconnected Graphs

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(This talk is based on joint work with Prof. N. Soltankhah and Prof. K. Khashyarmanesh.)

Abstract

Let G be a simple graph with vertex set $V(G)$. A coloring (or labeling) of a graph G is a partition of the vertices of G into classes, called the color classes. If a coloring contains exactly n disjoint non-empty color classes, then it is called an n-coloring. We say that a coloring with color classes $\{V_1, \ldots, V_\ell\}$ of G is distinguishing labeling if there is no non-trivial automorphism f of G with $f(V_i) = V_i$ for all $i = 1, \ldots, \ell$. We denote the minimum such ℓ by $D(G)$ and is called distinguish number of G. A distinguishing labeling is distinguishing coloring (or proper distinguishing coloring) if it provides a proper coloring for G. The distinguishing chromatic number of a graph G, denoted by $\chi_D(G)$, is the minimum ℓ such that $\{V_1, \ldots, V_\ell\}$ is distinguishing coloring. In 2006, Collins and Trenk proposed this coloring and called *proper distinguishing coloring* (or *distinguishing coloring*) [2]. This coloring has attracted the attention of researchers in a short period of time and many articles have been published about it. In [3], Harary, Hedetniemi, and Robinson introduced and studied the uniquely colorable graphs. In their work 'coloring' means that 'proper coloring'.

We say that a graph is uniquely distinguishing n-colorable if it has exactly one distinguishing n-coloring. Furthermore, we say a graph is uniquely distinguishing colorable if there is only one partition of its vertex set into the smallest possible number of distinguishing color classes. Actually a uniquely distinguishing colorable graph is a uniquely distinguishing $\chi_D(G)$-colorable. The symbols of distinguishing coloring of G will always denote $[\chi_D(G)]$. Any unexplained basic definitions in graph theory comes from [1].

In this talk, we present some results on uniquely distinguishing graphs, because of their applications in computing the distinguishing chromatic number of disconnected graphs. For this propose, we introduce two families of uniquely distinguishing colorable graphs, named them type 1 and type 2, and show that every disconnected uniquely distinguishing colorable graphs are the union of two isomorphic graphs of type 2. Also, we review the characterization of all graphs G of order n with property that $\chi_D(G \cup G) = \chi_D(G) = k$, where $k = n - 1, n$.

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Edge Locating Coloring of Graphs

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(This talk is based on joint work with Prof. Edy Tri Baskoro, Prof. D.A. Mojdeh and Prof. A. Erfanian.)

Abstract

In 2002, Chartrand et al introduced a coloring that we know as *locating coloring* [1]. In this coloring, the goal is to distinguish the vertices of a graph by their distance from a partition of the vertex set. Here, our goal is to distinguish the vertices of a connected graph by the distance of the matchings that partition the edge set. An edge locating coloring of a simple connected graph G is a partition of its edge set into matchings such that the vertices of G are distinguished by the distance of the matchings. The minimum number of the matchings of G that admitting an edge locating coloring is the edge locating chromatic number of G, and denoted by $\chi'_L(G)$. In fact, we can see this definition as the edge version of the locating coloring.

In this talk we initiate to introduce the concept of edge locating coloring, and determine of exact value $\chi'_L(G)$ of some custom graphs. The graphs G with $\chi'_L(G) \in \{2, m\}$ are characterized where m is the size of G. We investigate the relationship between order, diameter and edge locating chromatic number of G. For a complete graph K_n, we obtain the exact value of $\chi'_L(K_n)$ and $\chi'_L(K_n - M)$ where M is a maximum matching, indeed this result is also extended for any graph. We will determine the edge locating chromatic number of join graph $G + H$, where G and H are some well known graphs. In particular, for any graph G, we show that, there exists a relationship between $\chi'_L(G+K_1)$ and $\Delta(G)$. We investigate the edge locating chromatic number of trees and present a characterization bound for any tree in terms of maximum degree, leaves and the support vertices of trees. Moreover, we prove that any edge locating coloring of a graph is an edge distinguishing coloring. Finally, we present some open problem.

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Extending the Chain Theorem from Matroids to Internally 4-Connected Graphs

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(This talk is based on joint work with Prof. Guoli Ding, Louisiana State University.)

Abstract

Graph theory and matroid theory are interconnected because matroids provide a way to generalize and analyze the structural and independence properties found in graphs. Chun, Mayhew, and Oxley [1] have proven a chain theorem for internally 4-connected binary matroids and have provided a detailed analysis of the operations to produce such matroids. The primary focus of our study is to extend this matroid result to internally 4-connected graphs.

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Turan problem for graphs arising from geometric shapes

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(This talk is based on joint work with Jun Gao, Oliver Janzer, and Zixiang Xu.)

Abstract

While Turán type problem is the most studied topic in extremal combinatorics, some of the most basic bipartite degenerate Turán problems remain elusive. In this talk, I will discuss some recent advancements on this topic and new results on bipartite graphs arising from geometric shapes and periodic tilings commonly found in nature, including even prisms, planar hexagonal tiling and quadrangulations of plane, cylinder and torus.

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(H_1, H_2)-SUPERMAGIC SHACKLES

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Abstract

Let H_1 and H_2 be non-isomorphic graphs. A graph G is said to admit (H_1, H_2)-*covering* if every edge $e \in E(G)$ is contained in either a subgraph H_1 or H_2 of G. Let $f : V(G) \cup E(G) \to [1, |V(G)| + |E(G)|]$ be a bijection of a graph G. The labeling f is called (H_1, H_2)-*magic labeling* if G admits (H_1, H_2)-covering and there exists magic constants m_1 and m_2 such that

$$w(H_1) = \sum_{v \in V(H_1)} f(v) + \sum_{e \in E(H_1)} f(e) = m_1$$

for every subgraph H_1 of G and

$$w(H_2) = \sum_{v \in V(H_2)} f(v) + \sum_{e \in E(H_2)} f(e) = m_2$$

for every subgraph H_2 of G. Moreover, an (H_1, H_2)-magic labeling f is (H_1, H_2)-*supermagic labeling* if $\{f(v) \mid v \in V(G)\} = [1, |V(G)|]$. The graph G is (H_1, H_2)-*(super)magic* if G admits (H_1, H_2)-(super)magic labeling. In this paper, we present some new (H_1, H_2)-magic graphs.

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Some constructions of Ramsey $(C_4, K_{1,n})$-minimal graphs

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(This talk is based on joint work with Hilda Assiyatun and Edy Tri Baskoro.)

Abstract

For any simple graphs F, G and H, let us define $F \to (G, H)$ if for any red-blue coloring on the edges of graph F, there exists either a red copy of G or a blue copy of H. A graph F is called a Ramsey graph for (G, H) if $F \to (G, H)$. Additionally, if the graph F satisfies that $F - e \nrightarrow (G, H)$ for any edge e of F, then F is called a Ramsey (G, H)-minimal graph. The set of all Ramsey (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$. The most recently related results about some classes of graphs that belong to $\mathcal{R}(C_4, K_{1,n})$ were already done by Nabila et al. [1] that use paths to construct the theta-path graphs and Assiyatun et al. [2] that use any tree to construct the theta-tree graphs. In this research, we construct some Ramsey $(C_4, K_{1,n})$-minimal graphs that are based on a unicylic graph for any $n \geq 2$.

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On D-Distance Magic and Antimagic Labelings of Shadow Graphs

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Abstract

Let G be a graph with vertex set $V(G)$ and diameter $diam(G)$. Let $D \in \{0, 1, 2, 3, \ldots, diam(G)\}$ and g be a bijection from $V(G)$ to $\{1, 2, 3, \ldots, |V(G)|\}$. For two vertices $u, v \in V(G)$, the distance between u and v is denoted by $d(u, v)$. The *D-neighborhood* of a vertex $v \in V(G)$ is denoted and defined by $N_D(v) = \{u \in V(G) : d(u, v) \in D\}$ and its *D-weight* is $w_D(v) = \sum_{u \in V_D(v)} g(u)$. If $w_D(v)$ is a constant for every vertex $v\ V(G)$, then the graph G is called *D-distance magic* and g is called a *D-distance magic labeling* of G. If $w_D(v) \neq w_D(u)$ for every $u, v \in V(G)$, then G is called *D-distance antimagic*. In particular, if $\{wD(v) : v \in V(G)\}$ is a set $\{a, a + d, a + 2d, \ldots, a + (|V(G)| - 1)d\}$, where $a > 0$ and $d \geq 0$ are fix integers, then G is called (a, d)-*D-distance antimagic*. In these cases, g is called a *D-distance antimagic labeling* and an (a, d)-*D-distance antimagic labeling* of G, respectively. In this talk, we give some necessary conditions for shadow graph of a regular graph to be D-distance magic as well as (a, d)-D-distance antimagic. Also, we prove the existence and nonexistence of the D-distance magic labeling and the (a, b)-D-distance antimagic labeling of shadow graph of cycles and complete bipartite graphs for $D = \{1\}$, $\{0, 1\}$, $\{2\}$, and $\{0, 2\}$.

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An improved spectral lower bound of treewidth

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(This talk is based on joint work with Tatsuya Gima, Tesshu Hanaka, Hirotaka Ono, Yota Otachi.)

Abstract

In this talk, we improve the lower bound of treewidth shown by Chandran and Subramanian [1]. Let $\mathrm{tw}(G)$, Δ, λ denote the treewidth, the maximum degree, and the second smallest eigenvalue of the Laplacian matrix of a graph G with n vertices, respectively. Chandran and Subramanian [1] show that $\mathrm{tw}(G) \geq \frac{3\lambda n}{4\Delta+8\lambda} - 1$. We show that

$$\mathrm{tw}(G) \geq \frac{\lambda n}{\Delta + \lambda} - 1,$$

using the balanced-separator technique for treewidth computation by Robertson and Seymour [3] and properties of the Laplacian matrix. Previously, we presented a weaker improvement [2], which shows that $\mathrm{tw}(G) \geq \min\{\frac{3\lambda n}{4\Delta+3\lambda}, \frac{4\lambda n}{4\Delta+6\lambda}\} - 1$. The new lower bound is almost tight in the sense that for an infinite class of graphs the bound is only 1 smaller than the actual treewidth.

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On Quadratic Embedding Constants of Graphs

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Abstract

Let $G = (V, E)$ be a (finite connected) graph and $D = [d(x, y)]_{x,y \in V}$ its distance matrix. It is well known, tracing back to Schoenberg (1935-37), that G admits a quadratic embedding in a Euclidean space, i.e., there exists a map $\psi : V \to \mathbb{R}^N$ such that $\|\psi(x) - \psi(y)\|^2 = d(x, y)$ for $x, y \in V$, if and only if D is conditionally negative definite. This motivated us to define the *quadratic embedding constant* (*QE constant* for short) of G by

$$\mathrm{QEC}(G) = \max\{\langle f, Df \rangle \,;\, f \in C(V),\ \langle f, f \rangle = 1,\ \langle \mathbf{1}, f \rangle = 0\},$$

where $C(V)$ is the space of column vectors indexed by V, and $\mathbf{1}$ is the one of which entries are all one. Classification of graphs by means of QE constants is a relatively new challenge since the concept was introduced in Obata–Zakiyyah (2018) and Obata (2017). In this talk we will report some recent results in particular, on non-QE graphs and show some questions.

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Magic Labeling on Graphs with Ascending Subgraph Decomposition

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(This talk is based on joint work with Rinovia Simanjuntak and Saladin Uttungadewa.)

Abstract

Let G be a simple and finite graph of positive size q and let t be a positive integer with $\binom{t+1}{2} \leq q < \binom{t+2}{2}$. G is said to have an *ascending subgraph decomposition (ASD)* if G can be decomposed into t subgraphs $H_1, H_2, \ldots, H_t$ without isolated vertices where H_i is isomorphic to a proper subgraph of H_{i+1} for $1 \leq i \leq t-1$. (Alavi *et. al.*, 1987)

In this talk, we introduce a new type of magic labeling. Let G admit an ASD with subgraphs $H_1, H_2, \ldots, H_t$ and f a bijective mapping from $V(G) \cup E(G)$ to $\{1, 2, \ldots, |V(G)| + |E(G)|\}$. The weight of a subgraph H_i $(1 \leq i \leq t)$ is defined as $w(H_i) = \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e)$. If all subgraphs have the same weight, i.e., there exists a positive integer k such that $w(H_i) = k$ $\forall i \in [1, t]$, then f is called a magic ASD labeling for G. Additionally, we characterize complete graphs, stars, paths, and cycles admitting magic ASD labelings.

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All trees of radius two with distinguishing number three

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Abstract

Let $G(V, E)$ be a simple connected graph with the vertex-set V and the edge-set E. A vertex k-labeling on G is an onto mapping $f : V(G) \to \{1, 2, \cdots, k\}$. The *distinguishing number* of G, denoted by $D(G)$, is the least natural number k such that G has a vertex k-labeling that is preserved only by the *trivial automorphism.* In this talk, we will show among all trees of radius two, there are 1552 trees having distinguishing number three.

Keywords: *distinguishing number, graph automorphism, labeling, tree*

On Rainbow and Strong Rainbow Coloring on Two Classes of Windmill Graphs

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Abstract

An edge-coloring of a graph G is called a rainbow if any two vertices are connected by a path consisting of edges of different colors. The strong rainbow connection number is an extension of the rainbow connection numbers, where it refers to the shortest path, commonly known as the geodesic path. If G is a connected graph and every pair of vertices in G has a geodesic path whose edges are not the same color, G is strongly rainbow-connected. The rainbow and strong rainbow connection numbers of graph G, denoted by rc(G) and src(G), respectively, are the minimum number of colors needed to make G rainbow and strongly rainbow-connected. This study is interesting, and recently, many papers have been published about it. Some previous results only gave the lower and upper bound of rc(G) and src(G). Thus, finding an exact value of rc(G) and src(G) is significantly challenging. In this paper, we determine the exact values of rainbow and strong rainbow connection number of two classes of windmill graphs such as French windmill graph and Dutch windmill graph, and also certain generalizations of these graph.

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On the Edge Irregularity Strength of Some Trees

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Abstract

Let $G = (V(G), E(G))$ be a graph and k be a positive integer. A vertex k-labeling $f : V(G) \rightarrow \{1, 2, \ldots, k\}$ is called an edge irregular labeling if there are no two edges with the same weight, where the weight of an edge uv is $f(u) + f(v)$. The edge irregularity strength of G, denoted by $es(G)$, is the minimum k such that G has an edge irregular k-labeling. This labelings were introduced by Ahmad, Al-Mushayt, and Bača in 2014. In this paper, we determine the edge irregularity strength of banana tree and disjoin union of firecracker graphs.

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On $L(2,1)$-labeling of the comb product with a complete graph

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Abstract

An $L(2,1)$-*labeling* of a graph G is a function $f : V(G) \to \{0, 1, 2, \ldots, k\}$ such that every two distinct vertices $u, v \in V(G)$ satisfies $|f(u) - f(v)| \geq 2$ if their distance is 1, and $|f(u) - f(v)| \geq 1$ if their distance is 2. The smallest number k such that G admits an $L(2,1)$-labeling, is called the $L(2,1)$-labeling number of G. In this paper, we consider the comb product graphs. Let G and H be connected graphs, and $o \in V(H)$. The *comb product* between G and H at vertex o, denoted by $G \rhd_o H$, is a graph obtained from a copy of G and $|V(G)|$ copies of H, then identifying the vertex o of the i-th copy of H to the i-th vertex of G. In this paper, we provide the sharp general bounds of the $L(2,1)$-labeling number of $G \rhd_o K_n$ for any connected graphs G. We also determine an exact value of $L(2,1)$-labeling number of $G \rhd_o K_n$ for some families of graph G, including paths, stars, and complete graphs.

On Ramsey numbers for trees versus fans of even order

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(This talk is based on joint work with Suhadi Wido Saputro and Edy Tri Baskoro.)

Abstract

Given two graphs G and H. The least natural integer r such that any graphs F on r vertices satisfy either F has a copy of G or $\overline{F}$ contains a copy of H, is known as the graph Ramsey number $R(G,H)$. In this talk, we study the graph Ramsey numbers $R(T_n, F_{1,m})$ where T_n is a tree of order n, which is not a star, having maximum degree at least $n-3$, and $F_{1,m}$ is a fan of order $m+1$, with odd m.

On the study of influence and dominance indices rising from the species-habitat network

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Abstract

The species-habitat network (JSH) concept was introduced by Marini et al. (2019). The network has two sets of vertices: species and habitat. The edge on the network is defined as the occurrence of an animal in a location. The adjacency matrix of the network is called the species-habitat matrix. Two matrices related to ecology are constructed from this matrix: the species interaction matrix and the habitat occupancy matrix. Two matrices related to ecology are formed from this matrix: the species interaction matrix and the habitat occupancy matrix. Next, influence and dominance indices are obtained from these two matrices. We use these concepts to examine our dataset from the Bungo Area, Kerinci Seblat National Park.

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Distance-Local Rainbow Connection Number of Amalgamation of Cycles

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(This talk is based on joint work with Laurensius Fabianus Steven, Gabriella Aileen Mendrofa, Peter John, Novi Herawati Bong.)

Abstract

A rainbow geodesic in an edge-colored graph is the shortest path with all edges having different colors. A graph G is strongly rainbow connected if a rainbow geodesic connects every pair of vertices in G. The strong rainbow connection number of $G, src(G)$, is the minimum number of colors needed to make G strongly rainbow connected. Let d be a positive integer, a graph G is a $d-$local strong rainbow connected if a rainbow geodesic connects every pair of vertices of distance up to d. The $d-$local strong rainbow connection number of $G, lsrc_d(G)$, is the minimum number of colors in the $d-$local strong rainbow coloring of G. In this talk, we present the $d-$local strong rainbow connection number of amalgamation of cycles.

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Is it Possible to Have Constant Modular Irregularity Strength of Graph

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(This talk is based on joint work with M. Augurius.)

Abstract

In this talk, we consider a simple and finite graph G with order n. In 1988, Chartrand *et al.* defined an irregular labeling as an edge k-labeling $f : E(G) \to \{1, 2, \ldots, k\}$, for a positive integer k, where the vertex weights are different for all vertices. The vertex weight for a vertex v is the sum of all edge labels which is incidence to v. The minimum number k for this labeling is called the irregularity strength of a graph G and is denoted by s(G). In 2020, Bača *et al.* introduced the variation of irregular labeling in modular version. Modular irregular labeling of a graph G is an edge k-labeling $f : E(G) \to \{1, 2, \ldots, k\}$ such that the modular weight, which is defined by $wt_f = \Sigma_(v \in N(u))f(uv)(mod\ n)$, of all vertices are all different. The modular irregularity strenght of a graph G, denoted by ms(G), is a minimum number k such that a graph G has modular irregular labeling with the largest label k. In this talk, we discuss the possibility that a graph can have a constant modular irregularity strength.

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Counterexamples to the total vertex irregularity strength's conjectures

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(This talk is based on joint work with Rinovia Simanjuntak and Edy Tri Baskoro.)

Abstract

For a simple graph $G(V, E)$ and a positive integer k, a vertex irregular total k-labeling of G is a mapping $\varphi : V \cup E \to \{1, 2, \ldots, k\}$ such that $wt(x) \neq wt(y)$ for any two distinct vertices $x, y \in V$, where $wt(x) = \varphi(x) + \sum_{xz \in E} \varphi(xz)$. The minimum k for which G has a vertex irregular total labeling is called the total vertex irregularity strength of G and it is denoted by $\text{tvs}(G)$. Finding the total vertex irregularity strength of arbitrary graphs is a difficult and challenging problem; see [1, 2, 4, 5, 6, 7] for a few results on this topic.

In 2010, Nurdin, Baskoro, Salman and Gaos [3] posed two conjectures regarding the total vertex irregularity strength of trees and general graphs as follows: (i) for every tree T, $\text{tvs}(T) = \max\{\lceil (n_1+1)/2 \rceil, \lceil (n_1+n_2+1)/3 \rceil, \lceil (n_1+n_2+n_3+1)/4 \rceil\}$, and (ii) for every graph G with minimum degree δ and maximum degree Δ, $\text{tvs}(G) = \max\{\lceil (\delta + \sum_{j=1}^{i} n_j)/(i+1) \rceil : i \in [\delta, \Delta]\}$, where n_j denotes the number of vertices of degree j.

In this talk, we disprove the above-mentioned conjectures by giving infinite families of counterexamples.

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Set Colorings of the Cartesian Product of Some Graph Families

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(This talk is based on joint work with Janree Ruark C. Gatpatan and Timothy Robin Y. Teng both from Ateneo de Manila University.)

Abstract

In graph theory, one area that has received much attention from researchers is the area of neighbor-distinguishing colorings (i.e., colorings that induce a proper vertex coloring) [1, p. 756]. One such coloring is the set coloring [2]. For a nontrivial graph G, let $c : V(G) \to \mathbb{N}$ and define the *neighborhood color set* $NC(v)$ of each vertex v as the set containing the colors of all neighbors of v. The coloring c is called a *set coloring* if $NC(u) \neq NC(v)$ for every pair of adjacent vertices u and v of G. The minimum number of colors required in a set coloring is called the *set chromatic number* of G and is denoted by $\chi_s(G)$. In recent years, set colorings have been studied with respect to different graph operations such as join [3, 5], comb product [5], middle graph [4, 6], and total graph [7]. Continuing the theme of these previous works, we aim to investigate set colorings of the Cartesian product of graphs. In this work, we investigate the gap given by $\max\{\chi_s(G), \chi_s(H)\} - \chi_s(G \,\square\, H)$ for graphs G and H. In relation to this objective, we determine the set chromatic numbers of the Cartesian product of some graph families.

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No-three-in-line games on graphs

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(This talk is based on joint work with Sandi Klavžar and Jing Tian.)

Abstract

The *general position problem* originates with a puzzle of Dudeney on how to place pawns on a chessboard without three pawns being in a line. In the context of graph theory, we say that a set S of vertices of a graph G is in *general position* (or *no-three-in-line*) if no shortest path in G contains three vertices of S [2]. The *general position number* is the number of vertices in a largest general position set.

Inspired by the fields of game colouring and domination games, as well as the achievement game considered in [1], in this talk we discuss general position sets that are constructed through adversarial play. Two players, Builder and Blocker, take it in turns to choose vertices to add to a general position set S, always maintaining the no-three-in-line property. However, the goals of the players are diametrically opposed; Builder wishes to make the set S as large as possible, whilst Blocker wishes to minimise the size of S.

We determine the result of this game on graph families including trees, cycles, Kneser graphs and line graphs of complete graphs, and provide bounds and realisation results. In marked contrast to domination games, we also show that changing the order of the players can increase or decrease the size of the resulting set by an arbitrarily large amount.

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On the rainbow connection number of the connected inverse graph of a finite group

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Abstract

Let Γ be a finite group with $S_\Gamma = \{s \in \Gamma | s = s^{-1}\}$ and $T_\Gamma = \{t \in \Gamma | t \neq t^{-1}\}$. The inverse graph of a finite group Γ, denoted by $IG(\Gamma)$, is a graph whose set of vertices is Γ and two distinct elements $a, b \in \Gamma$ are adjacent if $ab \in T_\Gamma$. The rainbow connection number of a connected graph G, denoted by $rc(G)$, is the minimum number of colors needed to color the edges of G such that every two distinct vertices of G are connected by a path whose all edges are colored differently. In this paper, we discuss three aspects of the rainbow connection number of the connected inverse graph of a finite group. First, we propose a new upper bound for the rainbow connection number of the connected inverse graph of a finite group. It is known from [1] that the upper bound is $|T_\Gamma| + m + 2$, with $m = |\{s \in S_\Gamma | st = t^{-1}s$ for all $t \in T_\Gamma\}|$. In this paper, we propose $4 + m$ as a new upper bound. This is a better upper bound since $|T_\Gamma| \geq 2$ for any finite group Γ whose $IG(\Gamma)$ is connected. Second, we generalize the sufficient condition for a finite group Γ to have $rc(IG(\Gamma)) = 2$. It has been proven in [1] that for a finite group Γ, $rc(IG(\Gamma)) = 2$ if $st = ts$ for every $s \in S_\Gamma$ and every $t \in T_\Gamma$. In this paper, we prove that for a finite group Γ, $rc(IG(\Gamma)) = 2$ if $st \neq t^{-1}s$ for every $s \in S_\Gamma$ and every $t \in T_\Gamma$. At last, we show that for a finite group Γ and $k \geq 2$, if $rc(IG(\Gamma)) = k$, then every element of Γ can be expressed as a product of r elements of T_Γ, with $r \leq k$.

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Quiver Representation for Neural Network

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Abstract

A neural network can be seen as a directed graph with a loop, or we also call it a quiver. The quiver that is used for the neural network is called a quiver network. We will use a quiver arranged by layer to make a quiver network. A quiver network will be a neural network if we give a quiver representation. The quiver representation is a pair of a sequence of vector spaces and linear transformations. The vertices of the quiver will index the sequence of vector spaces, and the arrows will index the sequence of linear transformation in the quiver without the loops. Using quiver representation, we will see the morphism concept; then, we can form a moduli space. Using the moduli space, we can simplify the calculations in neural networks.

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On cyclic G-design where G is generalized petersen graph $P(4n, 2n1)$

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Abstract

It is known that an ordered ρ-labeling (also known as a ρ^+-labeling) of a bipartite graph G of size m can be used to obtain a cyclic G-decomposition of K_{2mt+1} for every positive integer t. We show that the generalized Petersen graphs $P(4n, 2n-1)$ admits a ρ^+-labeling for every positive integer n.

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Lower General Position Sets in Cartesian Products

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(This talk is based on joint work with James Tuite, Sharif Khudairi, Grahame Erskine.)

Abstract

The *general position problem* consists of finding a set S of vertices in a graph G such that, for any two vertices $u, v \in S$ no other vertex $w \in S$ lies on a shortest path between u and v. A *lower general position set* is a smallest general position set such that, if you try to add another vertex $z \in V(G) - S$ to the set S, S will no longer be in general position [1]. This is essentially the worst case outcome for a greedy algorithm when constructing a general position set.

Cartesian products are an active area of research in the general position problem (see for example [2] and [3]). In this talk we will discuss lower general position sets in Cartesian products of graphs. We investigate products of graph families including cycles, complete graphs and Kneser graphs and explore the relationship between the lower gp-number of the product and the lower gp-number of its factors.

To give bounds for the lower gp-number of any Cartesian product, it turns out to be useful to consider general position sets S with the special property that, for any vertex $u \in V(G) - S$, there is a shortest path in G containing u as an endpoint and two vertices of S; we call such a set a *terminal set*. It is not obvious that such a set always exists; however, we conjecture that every graph has a terminal set and present an algorithmic proof that graphs with diameter two or three have terminal sets, and also show that chordal graphs have terminal sets.

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Approximate Results for D-Antimagic Via Combinatorial Nullstelensatz

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Abstract

Let $G = (V(G), E(G))$ be a simple graph, D be a non-empty subset of $[0, diam(G)]$, D-neighborhood of a vertex x be $N_D(x) = \{y \in G | d(x, y) \in D\}$, and D-degree of a vertex x be $|N_D(x)|$. A D-antimagic injection f of a graph G is a map $f : V(G) \to [1, N]$ such that all D-vertex weights are different, where the D-vertex weight of a vertex x is $\Sigma_{y \in N_D(x)} f(y)$. For a particular graph class $\mathcal{X}$ of order n, the smallest possible N is called $Da(\mathcal{X}, n)$. In the case that $N = |V(G)|$, f is called a D-antimagic labeling of G. It is obvious that a graph containing two vertices with the same D-neighborhood does not admit a D-antimagic labeling. Recently, Simanjuntak et al. [3] conjectured that the converse of the previous statement is also true.

This talk uses Alon's Combinatorial Nullstelensatz [1] to provide partial evidence for Simanjuntak et al.'s conjecture. We prove the $\{1\}$-antimagicness of graphs with leaves, a generalization of a result for trees by Llado and Miller [2]. We also provide an upper bound for $Da(\mathcal{G}_{\Delta_D}, n)$, where $\mathcal{G}_{\Delta_D}$ is the graph class where Δ_D is the maximum D-degree. Finally, for two particular distance sets $D = \{i\}$ and $D = \{0, i\}$, we provide better upper bounds of $Da(\mathcal{X}, n)$, for a special graph class $\mathcal{X}$.

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Cutting to the Chase: Exploring the Use of Graph Cuts in DNA Splicing

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Abstract

The study of DNA splicing and its relationship with graph theory has gained increasing attention in genomics research. DNA splicing involves the fragmentation of genomes into smaller segments for analysis, while graph theory provides a mathematical framework for representing and analyzing complex relationships between objects. This research explores the intersection of DNA splicing and graph theory, with a focus on their applications in sequence analysis and genome assembly. We present a case study where we apply graph cutting techniques to a real-world DNA splicing dataset. The results demonstrate the utility of graph theory in resolving complex assembly problems, highlighting its potential for improving the accuracy and efficiency of genome assembly.

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Contributed Talks - Graph Algorithms

Algorithm for Describing The Terwilliger and Quantum Adjacency Algebras of a Distance-Regular Graph

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Abstract

In this talk we consider an algorithm for determining a basis for the Terwilliger and quantum adjacency algebras of a distance-regular graph with respect to some vertex x. For the Terwilliger algebra, we consider the generating set containing all nonzero $E^*_h A_i E^*_j$, where E^*_i and A_i respectively denote the i-th dual idempotent with respect to x and the i-th distance matrix. For the quantum adjacency algebra, we consider the generating set consisting of the raising, flat, and lowering matrices. The naive algorithm consists of iteratively taking the products of member pairs of the generating set and adding to the set any products not in its span, until a basis is obtained. We explore some optimizations to the algorithm, among them sorting the vertices of the graph according to the distance from x, which will produce generating matrices with a block-matrix structure reducing the number of matrix multiplications required.

HYBRID ANT COLONY OPTIMIZATION ALGORITHM WITH BINARY GRAY WOLF OPTIMIZATION FOR DETOUR METRIC DIMENSION AND BI-METRIC DIMENSION PROBLEM

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Abstract

In this paper, two types NP-hard optimization problems on graph are discussed: the detour metric dimension and the bi-metric dimension. Both are present in many diverse areas, including pattern recognition, monitoring the movement of robots on a network, and analyzing the structural properties of chemical structures. The metric dimension $dim(G)$ of graph G is the minimum number of vertices such that every vertex of G is uniquely determined by its vector of distances to the chosen vertices. This concept was developed into the detour metric dimension $D\beta(G)$ and the bi-metric dimension $\beta_b(G)$, by considering the detour distance of two vertices. To solve these two problems on large graphs, a computational approach is needed. Ant colony optimization, a probabilistic based metaheuristic algorithm is designed for finding the detour distance. Results prove the capabilities of the hybridation of ant colony optimization algorithm and the binary gray wolf optimization algorithm to search the detour metric dimension and the bi-metric dimension.

The Price of Stability of Fractional Hedonic Games on Graphs with Many Triangles [1]

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Abstract

We consider *fractional hedonic games*, a subclass of modeling coalition formation games on graphs based on individual preferences in which vertices represent agents and the weight of an edge (i, j) denotes the value that agent i has for agent j. In this work, we deal with fractional hedonic games on undirected and unweighted graphs, which are called *simple and symmetric fractional hedonic games* [1]. In fractional hedonic games, a coalition structure is represented by a partition of the vertices, where each set represents a coalition. Given a coalition structure, the utility of each agent is defined as the average weights of its incident edges. The sum of the utilities of each agent is called the social welfare. One of the socially desirable coalition structures is one that maximizes social welfare and such a coalition structure is said to be *social optimum*. On the other hand, a coalition structure is said to be *stable* when no agent can increase its utility by deviating to another coalition. Note that the social optimum is not necessarily stable. To measure the efficiency of a stable solution, the Price of Stability (PoS) is defined as the total utility of the social optimum divided by the total utility of the coalition structure that maximizes social welfare in stable coalition structures.

The previous studies imply that the existence of triangles might cause the gap between the lower bound and upper bound of PoS [2]. In this work, we consider two types of graph classes containing many triangles: split graphs and block split graphs. We give upper bounds of PoS on these graph classes. Here, a graph $G = (V, E)$ is called *block* if every biconnected component in G is a clique. A *split* graph is a graph whose vertex set can be partitioned into a clique K and an independent set I, such that each vertex in I is adjacent to only vertices in the clique K. A *block split* graph is split and block. We show the following theorems.

Theorem 1. *For any split graph G,* $\mathrm{PoS}(G) \le 0.70711\sqrt{n} + 1.2519$.

Theorem 2. *For any block split graph G,* $\mathrm{PoS}(G) \le 3$.

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A dynamic general position problem

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(This talk is based on joint work with Sandi Klavžar, James Tuite, Ismael Yero.)

Abstract

A subset S of vertices of a graph G is in general position if no shortest path between a pair of vertices in S contains a third vertex of S [1, 2]. S can be viewed as a set of robots standing on the specified vertices of G and sending signals to each other, such that each pair of robots can communicate without being intercepted by any other robot. This is a static system; in a more realistic setting, the robots would be able to move through the network.

Therefore we study the following dynamic version of the general position problem: how many robots may be assigned to the vertices of a graph G such that the robots can visit every vertex of the graph whilst always remaining in general position. The largest possible number of robots in such a configuration is called the *mobile position number* $\mathrm{Mob}_{\mathrm{gp}}(G)$. We give some bounds for the mobile position number in terms of other graph invariants and give exact values for families including block graphs, line graphs of complete graphs, unicyclic graphs and Kneser graphs. We close with some open problems.

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Enumerate All Routes on a Doughnut

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Abstract

In this talk we consider a very restricted version of the drawing problem. Given a matching $M = (U \cup V, E)$ as a bipartite graph, two concentric circles, the cyclic ordering of the vertices in U and the cyclic ordering of the vertices in V, we wish to draw M with the minimum number of edge crossings so that the vertices in U are on the smaller circle with the given cyclic ordering and the vertices in V are on the larger circle with the given cyclic ordering. We call the problem the doughnut routing problem. We design an $O(n^3)$ time algorithm to solve the problem. The main idea of the algorithm is a reduction to a set of the minimum length generator sequence problems. Moreover we propose an enumeration algorithm for optimal solutions of the doughnut routing problems by using Reverse-search algorithm. Our algorithm implicitly defines a tree structure for all optimal solutions then enumerates all optimal solutions based on the tree.
Keywords: doughnut routing problem, minimum length generator sequence problem

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An algorithm for the intersection problem on two poset matroids on a $(\mathbf{2}+\mathbf{1})$-free poset

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Abstract

The notion of poset matroids is one of generalizations of matroids. A poset matroid is a pair of a poset P and a collection $\mathcal{F}$ of ideals of P satisfying several conditions. The poset matroid intersection problem is a problem to find a largest common independent ideal for two poset matroids. While a good characterization for intersection of poset matroids was given, there is no known efficient algorithm to solve the poset matroid intersection problem. In our paper, we show that the poset matroid intersection problem can be solved efficiently for poset matroids on a $(\mathbf{2}+\mathbf{1})$-free poset.

1 Introduction and Preliminaries

The notion of poset matroids is one of generalizations of matroids ([1], [2]). Let $P = (E, \leq)$ be a poset. A subset $I \subseteq E$ is called an *ideal* of P if we have $x \in I$ whenever $y \in I$ and $x \in E$ with $x \leq y$. For a subset $X \subseteq E$, let $\mathrm{Min}(X)$ denote the set of all minimal elements of X in P.

Definition 1. *Let $P = (E, \leq)$ be a poset and $\mathcal{F}$ be a collection of ideals of P. The pair of $(P, \mathcal{F})$ is called a* poset matroid *if it satisfies the following conditions.*

(i) $\emptyset \in \mathcal{F}$.

(ii) *If $X \subseteq Y \in \mathcal{F}$ and X is an ideal of P, then $X \in \mathcal{F}$.*

(iii) *If $X, Y \in \mathcal{F}$ and $|X| < |Y|$, there exists an element $e \in \mathrm{Min}(Y \setminus X)$ such that $X \cup \{e\} \in \mathcal{F}$.*

For a poset matroid $(P, \mathcal{F})$, each ideal in $\mathcal{F}$ is said to be *independent* and each maximal independent ideal is called a *base*.

For two poset matroids $(P, \mathcal{F}_1)$ and $(P, \mathcal{F}_2)$, an element in $\mathcal{F}_1 \cap \mathcal{F}_2$ is called a *common independent ideal*. The *poset matroid intersection problem* is a problem to find a largest common independent ideal for two poset matroids. While a good characterization for the size of a largest common independent ideal was given in [6], any efficient algorithm for this problem is not known. Another generalization of the matroid intersection problem, called the poset matching problem, was studied in [4]. It is shown that the poset matching problem can be solved efficiently for a certain class of bipartite graphs in [4].

In our paper, we show that the poset matroid intersection problem can be solved efficiently for poset matroids on a certain poset.

2 Main Result

We give, first, the definition of $(\mathbf{2}+\mathbf{1})$-free posets.

Definition 2. *A poset is said to be* $(\mathbf{2}+\mathbf{1})$-free *if it does not contain* $\mathbf{2}+\mathbf{1}$ *as an induced subposet, where* $\mathbf{2}+\mathbf{1}$ *is the disjoint union of 2-element chain and an element.*

Our main result is as follows.

Theorem 1. *The poset matroid intersection problem can be solved in polynomial time for any two poset matroids on a* $(\mathbf{2}+\mathbf{1})$*-free poset.*

In our paper, we prove the theorem by using the fact that matroid intersection problem can be solved efficiently ([3], [5]).

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Fixed-Parameter Algorithms for Fixed Cardinality Graph Partitioning Problems on Sparse Graphs

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Abstract

For an undirected weighted graph $G = (V, E)$ and a vertex subset $S \subseteq V$, we define a function $\varphi_G(S) := (1 - \alpha) \cdot m(S) + \alpha \cdot m(S, V \setminus S)$, where $\alpha \in [0, 1]$ is a real number, $m(S)$ is the sum of weights of edges having two endpoints in S, and $m(S, V \setminus S)$ is the sum of weights of edges having one endpoint in S and the other in $V \setminus S$. Given an undirected weighted graph $G = (V, E)$ and a positive integer k, Max (Min) α-Fixed Cardinality Graph Partitioning (`Max (Min)` α-`FCGP`) is the problem to find a vertex subset $S \subseteq V$ of size k that maximizes (minimizes) $\varphi_G(S)$. This problem is a generalization of many NP-hard graph optimization problems such as Densest (Sparsest) k-Subgraph, Max (Min) k-Partial Vertex Cover, and Max (Min) $(k, n-k)$- Cut. For the parameterization by the solution size k plus degeneracy d, `Max` α-`FCGP` is known to be W[1]-hard for $\alpha \in [0, 1/3]$, but `Max (Min)` α-`FCGP` is fixed-parameter tractable for $\alpha \in [1/3, 1]$ in the maximization case and for $\alpha \in [0, 1]$ in the minimization case if an input graph is unweighted [?]. Panolan and Yaghoubizade propose a $2^{kd+k}(kd)^{O(\log(kd))}n^{O(1)}$-time algorithm for Max k-Partial Vertex Cover (equivalently, `Max 1/2-FCGP`) on weighted graphs [?].

In this talk, we show that `Max` α-`FCGP` for $\alpha \in [1/3, 1]$ and `Min` α-`FCGP` for $\alpha \in [0, 1/3]$ can be solved in time $2^{kd+k}(kd)^{O(\log(kd))}n^{O(1)}$ by extending the algorithm for Max k-Partial Vertex Cover proposed by Panolan and Yaghoubizade [?]. We also give a $2^{kd+O(k)}(kd)^{O(\log(kd))}n^{O(1)}$-time algorithm for the connected version of `Max (Min)` α-`FCGP`.

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Contributed Talks - Complexity and Winning Strategies of Puzzles and Games

COMPLEXITY OF HANANO AND JELLY-NO UNDER VARIOUS CONSTRAINTS

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Abstract

This work shows new results on the complexity of games Hanano and Jelly-No with various constraints on the width of the board and number of colours.

Hanano and Jelly-No are both one-player, 2D side-view puzzle games created by Qrostar and available online [1]. They consist of a dynamic board containing fixed platforms and coloured blocks which can be moved to the right or left by the player and are subject to gravity. The goal of both games is to move the coloured blocks in order to reach a specific configuration and make them interact with other elements of the game. In Hanano the goal is to make all the coloured blocks bloom by making contact with flowers of the same colour. In Jelly-No the goal is to merge all coloured blocks of a same colour, which also happens when they make contact.

Hanano was proven by Michael C. Chavrimootoo [2] to be PSPACE-Complete under the restriction that all movable blocks are the same colour. Jelly-No was proven by Chao Yang [3] to be NP-Complete under the same restriction and NP-Hard in the general case.

We show that this result holds under the restriction that the width of the board is limited to 5 columns, and that 1-colour Hanano is NP-Hard even when the width is limited to 6 columns. Finally, we show that Jelly-No is PSPACE-Complete with two colours and the use of black jellies.

A full version of this paper can be found on the following page:
`https://www.irif.fr/~vmitsou/jelly.pdf`

Keywords: Combinatorial games ; Complexity ; Hanano Puzzle ; Jelly-No Puzzle ; Motion planning ; NP-Hard ; PSPACE-Complete.

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Upper and Lower Bounds on the Optimal Questions for YOMEN

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Abstract

YOMEN is a 2-player 3D code-breaking game released in 2020. A code-breaking game is a game in which two players are divided into a code maker and a code breaker, and the code breaker tries to find the *code* defined by the code maker in a way appropriate to each game. Code-breaking games include Mastermind and Wordle, and the number of questions required to identify the code is well studied; e.g., Mastermind [1], generalized Mastermind [2], and Wordle [3]. In the formal rule of YOMEN, both of two players act as a code maker and a code breaker, and try to find the opponent's code, but in this study, we focus on the role of the code breaker for a given code. In YOMEN, a code is an *arrangement* of three blocks colored in red, yellow, and black on 3×3 cells, so that it satisfies certain rules. The code breaker can ask two types of questions, called *side view* about a row or column and *top view* about a cell. For the question about a side view, the code maker answers the colors of blocks on the corresponding row or column, and for the question about a top view, she answers the color of the top block with the height from the corresponding cell. In this study, we assume that the code breaker is not allowed to announce the arrangement by unconvinced guesswork, but only when the previous questions can identify the arrangement made by the code breaker. We say that an arrangement is identified when there is a unique arrangement satisfying answers to questions, and the optimal number of questions is the minimum number of questions required to identify an arbitrary legal arrangement. We obtain the following three theorems on the number of legal arrangements and the upper and lower bounds on the optimal number of questions for YOMEN.

Theorem 1. *The total number of legal arrangements in YOMEN is 19272.*

Theorem 2. *The optimal number of questions for YOMEN is at least 6.*

Theorem 3. *The optimal number of questions for YOMEN is at most 8.*

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Critical Sets of n-omino Sudoku

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Abstract

Sudoku, also called *Number Place*, is a pencil puzzle, where an instance is an incomplete 9×9 Latin square further partitioned into nine 3×3 subgrids. The digits put in advance are called *clues*. The goal of Sudoku is to complete a 9×9 Latin square, each subgrid of which contains all digits from one to nine, i.e., every digit occurs exactly once. Since one subgrid corresponds to one constraint, together with the constraints for the Latin square, the total number of constraints on Sudoku is $9+9+9 = 27$. In the study of Latin square completion, clues realizing the uniqueness of the solution with the minimum size is called a *critical set*. By defining a critical set of Sudoku similarly, we can say that the size of a critical set of Sudoku is 17. *Nonomino Sudoku* is a generalized variant of Sudoku, where a *nonomino* is a polyomino of order 9. The basic rule of Nonomino Sudoku is the same as Sudoku, but an instance is an incomplete 9×9 Latin square partitioned into 9 nonominoes, and the goal is to complete the Latin square such that each nonomino contains all digits.

In this study, we further generalize Nonomino Sudoku to n-omino Sudoku, which is defined on an $n \times n$ grid. We prove the following theorem.

Theorem 1. *The size of a critical set of n-omino Sudoku is $n - 1$.*

We introduce the notion of degeneracy for n-omino Sudoku. In Nonomino Sudoku, a nonomino can coincide with a column or row. Although $n \times n$ Sudoku as a generalization of the ordinary Sudoku has $3n$ constraints, the number of constraints of n-omino Sudoku can be smaller than $3n$ due to such coincidences, and then we say that some constraints *degenerate*. We define the *degeneracy* of an n-omino Sudoku instance as the number of nonominoes that coincide with a column or row. Thus, an n-omino Sudoku instance with degeneracy n is just an instance of the Latin square completion. It is known that the size of a critical set of $n \times n$ Latin square (i.e., n-omino Sudoku with degeneracy n) is at most $\lfloor n^2/4 \rfloor$. The following is a corollary of the above theorem for n-omino Sudoku with a small degeneracy.

Corollary 2. *There exists an n-omino Sudoku instance with degeneracy 0 or 1 such that its critical set has size $n - 1$.*

Intuitively, an instance with a small number of clues tends to have more solutions, or more clues tend to make a solution unique. Thus, we might expect that an instance with no clue has many solutions. However, there is an n-omino Sudoku instance with no clue that has no solution. More specifically, we have the following.

Theorem 3. *Let k and n be positive integers satisfying $n \leq 2k + 4$. Then, there is an n-omino Sudoku instance with degeneracy k and no clue such that it has no solution.*

On variations of Yama Nim ∼ What happens if you return tokens in Nim games? ∼

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(This talk is based on joint work with Takahiro Yamashita.)

Abstract

Yama Nim is a 2 heaps nim where players take more than 2 tokens from one heap, and return 1 token to the other heap. Triangular Nim is a generalization of Yama Nim, where the players are allowed to return any positive number of tokens, as far as the total number of the tokens decreases. Whoever removes the last token wins (normal play). Both Yama Nim and Triangular Nim have the same set of the $\mathcal{P}$-positios, namely $\{(x,y) \in \mathbb{Z}^2_{\geq 0} \mid -1 \leq x-y \leq 1\}$.

We also computed the Grundy numbers for these games for $\mathcal{N}$-Positions: For Yama Nim, if (x,y) is in the $\mathcal{N}$-Position (in other words, if $|x-y|>1$), the Grundy number is $\mathrm{Min}\,(x,y)+1$. The Grundy numbers for the Triangular Nim have more interesting behavior: When (x,y) is in the $\mathcal{N}$-position with $x<y$, for each Grundy number $g>0$, choose $d \in \mathbb{Z}_{>0}$ so that $\dfrac{d(d-1)}{2} \leq g < \dfrac{d(d+1)}{2}$, and consider the arithmetic progression

$$a_0 = g+1, a_1 = g+1+d, a_2 = g+1+2d, \ldots, a_k = g+1+kd, \ldots.$$

If $(x,y) = (a_k, a_{k+1})$ for some $k \in \{0,1,2,3,\ldots\}$, then the Grundy number for (x,y) is g. When (x,y) does not fit into any of these patterns, then the Grundy number for (x,y) is $x+y-1$.

We also considered a Wythoff twist of these Nim games. In Triangular α-Wythoff Nim (with $\alpha \in \mathbb{Z}_{\geq 0}$), the player is allowed to take tokens, either in the Triangular Nim way, or she/he can choose to take tokens from both heaps, at least 1 token from each, say $i>0$ tokens from the first heap and $j>0$ tokens from the second heap, as far as $|i-j| \leq \alpha$. The set of $\mathcal{P}$-positions for the Triangular 0-Wythoff Nim in $\{(x,y) \mid x \leq y\}$ is

$$\{(0,0),(0,1),(1,3),(3,6),(6,10),(10,15),(15,21),(21,28),(28,36),\ldots\},$$

in other words, triangular numbers $0,1,3,6,10,15,\ldots$ appear in the description of the set of $\mathcal{P}$-positions. Also for the $\mathcal{P}$-positions for the Triangular 1-Wythoff Nim in $\{(x,y) \mid x \leq y\}$ is

$$\{(0,0),(0,1),(1,4),(4,9),(9,16),(16,25),(25,36),(36,49),(49,64),\ldots\},$$

in other words, square numbers $0,1,4,9,16,25,36,\ldots$ appear. Similar patterns appear (Pentagonal numbers, Hexagonal numbers, ...) for $\alpha = 2,3,\ldots$.

We will also give more rules, with more sequences, or another new patterns, for variations of Yama Nim.

Grundy Numbers of Three-Dimensional Chocolate Bar Games

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Abstract

Chocolate-bar games are variants of the CHOMP game. A two-dimensional chocolate bar is a rectangular array of squares with some squares removed. There is a bitter square at the position $(0,0)$, and the height and the depth of the chocolate bar are defined by two functions f and g. Two players take turns and cut the chocolate bar along a horizontal or vertical line into two parts and eat that part that does not contain the bitter square. The player who leaves the opponent with the single bitter square is the winner. In prior work, the authors studied the two-dimensional case when $g(i) = 0$, and in the present article, they treat the case that $g(i)$ is not constantly zero. They characterize functions f, g such that the Sprague–Grundy value of $CB(f, g, x, y, z)$ is $x \oplus y \oplus z$. Then, they apply the results of two-dimensional chocolate bars to three-dimensional chocolate bars.

1 Introduction

Let $\mathbb{Z}_{\geq 0}$ be a set of non-negative numbers. Chocolate-bar games are variants of the CHOMP game. A two-dimensional chocolate bar is a rectangular array of squares in which some of the squares are removed. A bitter square printed in black is included in some part of the bar. Figures 1, 3, 4 display examples of two-dimensional chocolate bars. Each player takes their turn to break the bar in a straight line along the grooves into two parts, and eats the part without the bitter square. The player who leaves the opponent with the single bitter block (black block) is the winner.

A three-dimensional chocolate bar is a three-dimensional array of cubes in which a bitter cubic box printed in black is included in some part of the bar. Figures 5 and 7 display examples of a three-dimensional chocolate bar.

Each player takes their turn to cut the bar on a plane that is horizontal or vertical along the grooves into two parts, and eats the part without the bitter cubic box. The player who leaves the opponent with the single bitter cube is the winner. Examples of cut chocolate bars are depicted in Figures 8, 9, and 10.

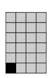

Figure 1:

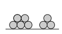

Figure 2:

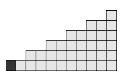

Figure 3:

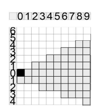

Figure 4:

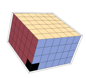

Figure 5:

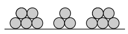

Figure 6:

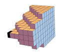

Figure 7:

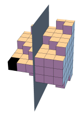

Figure 8:

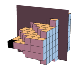

Figure 9:

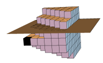

Figure 10:

Definition 1. *Let g, h be monotonically increasing functions. A two-dimensional chocolate bar is a rectangular array of squares with some squares removed. There is a bitter square at the position* $(0, 0)$. *There are $z + 1$ columns of squares, and for $i \in \mathbb{Z}_{\geq 0}$ such that $i \leq z$, the height of the i-th column is given by* $\min(g(i), x) + 1$, *and the depth of the i-th column is given by* $\min(h(i), y) + 1$. *We denote this chocolate bar by $CB(g, h, x, y, z)$.*

The most simple two-dimensional chocolate bar is a rectangular bar of chocolate with a bitter corner, as shown in Figure 1. Because the horizontal and vertical grooves are independent, an $m \times n$ rectangular chocolate bar is similarly structured as the game of Nim, which includes heaps of $m - 1$ and $n - 1$ stones. Therefore, the chocolate-bar game (Figure 1) is mathematically the same as Nim, which includes heaps of 5 and 3 stones (Figure 2). Because the Grundy number of the Nim game with heaps of $m-1$ and $n - 1$ stones is $(m - 1) \oplus (n - 1)$, the Grundy number of this $m \times n$ rectangular bar is $(m - 1) \oplus (n - 1)$.

Therefore, it is natural to search for a necessary and sufficient condition, wherein a chocolate bar may have a Grundy number calculated using the Nim-sum as the height and width of the two-dimensional chocolate bars.

We have already presented the necessary and sufficient conditions in [1] when the depth of chocolate bar is zero. In the present article, we study chocolate bars when the depth is not zero, such as Figure 4 and Figure 7.

For any position $\mathbf{p}$ of game $\mathbf{G}$, there is a set of positions that can be reached by precisely one move in $\mathbf{G}$, which we denote as $move(\mathbf{p})$.

Let $\mathbf{p}$ be a position of an impartial game. The associated *Grundy number* is denoted by $G(\mathbf{p})$, and is recursively defined by $G(\mathbf{p}) = mex\{G(\mathbf{h}) : \mathbf{h} \in move(\mathbf{p})\}$.

For any position $\mathbf{g}$ of $\mathbf{G}$, $G_{\mathbf{G}}(\mathbf{g}) = 0$, if and only if $\mathbf{g}$ is a $\mathcal{P}$-position. Therefore, Grundy numbers are an important research topic in combinatorial game theory.

Definition 2. *A monotonically increasing function h is said to have the NS property if* $\left\lfloor \frac{z}{2^i} \right\rfloor = \left\lfloor \frac{z'}{2^i} \right\rfloor$ *for some z, $z' \in \mathbb{Z}_{\geq 0}$, and some natural number i, then* $\left\lfloor \frac{h(z)}{2^{i-1}} \right\rfloor = \left\lfloor \frac{h(z')}{2^{i-1}} \right\rfloor$.

Theorem 1. *Let g, h be monotonically increasing functions and $\mathcal{G}_{g,h}(x, y, z)$ be the Grundy number of $CB(g, h, x, y, z)$. Then g, h satisfy NS property in Definition 2 if and only if* $\mathcal{G}_{g,h}(x, y, z) = x \oplus y \oplus z$.

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A Variant of Restricted Nim

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Abstract

The authors present formulas for the winning positions of the previous player's positions of a variant of restricted Nim. In this study, we investigate the case that in k-th turn, you can remove $f(k)$ stones at most, where f is a function whose values are natural numbers.

1 Introduction and a Restricted Nim

The authors present formulas for the winning positions of the previous player's positions of a variant of Maximum Nim. Let $\mathbb{Z}_{\geq 0}$ and $\mathbb{N}$ represent the sets of non-negative integers and natural numbers, respectively.

The classic game of Nim is played with stone piles. A player can remove any number of stones from any one pile during their turn; the player who takes the last stone is considered the winner.

There are many variants of the classical game of Nim. In Maximum Nim, we place an upper bound $f(n)$ on the number of stones that can be removed in terms of the number n of stones in the pile. As for the research of Maximum Nim, see [1].

In this study, we investigate the case that in k-th turn, you can remove $f(k)$ stones at most, where f is a function whose values are natural numbers. This seems to be a new restriction on the number of stones to be taken.

Definition 1. *Let $m \in \mathbb{N}$ and f is a function whose values are natural numbers. Suppose there is a pile of stones, and two players take turns removing stones from the pile. In kth turn, the player is allowed to remove at least one stone and at most $f(k)$ stones. The player who removes the last stone is the winner.*

The restricted nims that we study in the present article are impartial games without draws, there will be only two kinds of positions.

Definition 2. (a) *A position is referred to as a $\mathcal{P}$-position if it is a winning position for the previous player (the player who just moved), as long as he/she plays correctly at every stage.*
(b) *A position is referred to as an $\mathcal{N}$-position if it is a winning position for the next player, as long as he/she plays correctly at every stage.*

Definition 3. *We denote by (n, k) the position of the game when a player removes a stones in the k-th turn and the number of stones is n,*

Definition 4. *For $u \in \mathbb{N}$, the set of all the positions that can be reached from position (u, k) is defined as move(u, k). For any $t \in Z_{\geq 0}$, we have move$(u, k) = \{(u - t, k + 1) : t \in \mathbb{N}$ and $1 \leq t \leq \min(u, f(k))\}$.*

1.1 When $f(k) = mk$ for a fixed natural number m

When $f(k) = mk$ for a fixed natural number m, we can describe the set of $\mathcal{P}$-positions by Theorem 1.

Theorem 1. *For $n \in \mathbb{Z}_{\geq 0}$, let $\mathcal{P}^m_{k,n} = \{n(mn+m(k-1)+1)+i, k) : i \in \mathbb{Z}_{\geq 0}$ and $i \leq mn\}$, $\mathcal{P}^m_k = \cup\{\mathcal{P}_{k,n} : n \in \mathbb{Z}_{\geq 0}\}$, and $\mathcal{P}^m = \cup\{\mathcal{P}^m_k : k \in \mathbb{N}\}$. Then, $\mathcal{P}^m$ is the set of $\mathcal{P}$-positions.*

The graphs of the case when $f(k) = k$ and $f(k) = 4k$ are presented in figures 1 and 2. For the list (n, k) in the graphs, the horizontal coordinate n is for the number of stones, and the vertical coordinate k shows that the player removes stones at kth turn from this position.

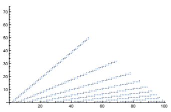

Figure 1:

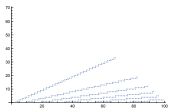

Figure 2:

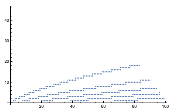

Figure 3:

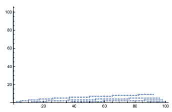

Figure 4:

1.2 Cases for other types of functions.

The authors have calculated the sets of $\mathcal{P}$-positions for various types of functions $f(k)$ by computers. If we compare graphs in figures 1, 2, 3 and 4, it seems that graphs look similar when $f(k)$ is a polynomial of k.

When $f(k) = \lfloor log_2 k \rfloor + 1$ or $f(k) = \lfloor log_{10} k \rfloor + 1$, we get an interesting graph. See figures 5 and 6. When $f(k)$ is the k-th number of Finonacci sequence or Tribonacci sequence, we also get interesting graphs! See figures 7 and 8. The authors have not discovered any formula that describes these sets of $\mathcal{P}$-positions.

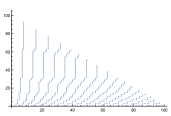

Figure 5:

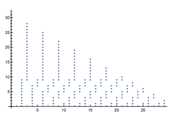

Figure 6:

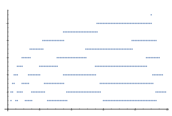

Figure 7:

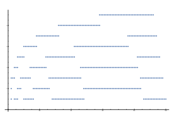

Figure 8:

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A Study on Sentiment Analysis of Public Response to the New Fuel Price Policy in 2022: A Support Vector Machine Approach

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Abstract

The public's reaction to the government's decision to increase fuel prices as a consequence of the surge in global crude oil prices has generated significant interest and debate. Analyzing the classification of public responses to a policy is essential as it provides insights into identifying the most appropriate timing for implementing policies while minimizing negative reactions. Therefore, motivated by [1] this study aims to apply the Support Vector Machine (SVM) algorithm to classify public sentiments in response to the new fuel price in 2022. The data used in this research were collected from Twitter using web scraping techniques, specifically leveraging the Python library, snscrape. The scraped data is preceded by a text preprocessing stage before it can be used in the classification model development. The model was built using Python in the Google Collaboratory Integrated Development Environment (Google Collab IDE), and the SVM algorithm was applied to categorize public opinions into positive (+) or negative (-) responses. The resulting classification model was subjected to validation testing, employing the confusion matrix method [2], which yielded an accuracy rate of 81%. The analysis indicated that 63.55% of the public had a negative (-) response, while 36.45% expressed a positive (+) sentiment toward the government's policy. Furthermore, the study revealed a relationship between the number of iterations and the model's accuracy, with increasing iterations leading to a convergence toward 81%. The research findings were visualized using Word Clouds, Pie Charts, and a simple Graphical User Interface (GUI) for user accessibility. Considering the large proportion of negative responses from the public to the government's decision to raise fuel prices at that time, it became evident that it was not the opportune moment for implementing the policy. The government's repeated delays in executing the new fuel price in 2022 demonstrate their consideration of the appropriate timing, aiming to minimize negative public reactions. Thus, in the future studies focusing on the classification of public responses will be strategic as the study playing an integral role in Decision Support Systems (DSS) for making timely and well-informed decisions.

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Toichika is NP-Complete

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Abstract

Toichika is a pencil-and-paper logic puzzle first published by a Japanese company Nikoli. The puzzle consists of a rectangular grid divided into polyominoes called regions, with some cells already containing an arrow. The player has to put arrows pointing in horizontal or vertical direction into cells according to the following rules.

1. Each region contains exactly one arrow.
2. Two arrows pointing towards each other with no other arrow between them form a pair; all arrows must be paired.
3. The paired arrows cannot be in horizontally or vertically adjacent regions.

We show that the problem of deciding whether a given Toichika instance has a solution is NP-complete by a reduction from the 3-SAT problem.

Categories of Games: A Survey

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Abstract

In this talk, we will take a look at various category of games that has been proposed so far from category of open games ([1], [2], [3]) to a category of graphs generated from games ([4]).

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Partizan Restricted Chocolate Games

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Abstract

In this paper, we consider partizan restricted chocolate bar games. In partizan restricted chocolate bar games, there are players designated as Left and Right and chocolate bars with black and white stripes. Suppose that they have a chocolate bar with n black square and m white square. Left cuts the chocolate bar in two parts, and she can eat the part with equal to or less than $\lceil \frac{n}{2} \rceil$ black squares. Similarly, Right cuts the bar and eats the part with equal to or less than $\lceil \frac{m}{2} \rceil$ white squares. A player loses when they cannot eat the remaining chocolate bar. We provide formulas that describe the winning positions of the previous player, Right, and Left players. We also present a conjecture on three-dimensional partisan chocolate bar game.

1 Introduction

In the present work, we consider a partisan chocolate bar game with restrictions on the number of black or white squares to be eaten. We denote the set of natural numbers as $\mathbb{N}$.

There are other types of restricted chocolate games, where the color of the chocolate is the same for each square. See [1]

Let the name of two players be Left (using she as a pronoun) and Right (using he as a pronoun). Here we use chocolate bars with black and white stripes. See Figures 1, 2, 3, and 4 for examples. The use of this black and white chocolate bar was suggested by Prof. R. J. Nowakowski when one of the authors discussed a partizan version of chocolate bar games at Combinatorial Game Theory Colloquium IV.

Suppose that they have a chocolate bar with n black square and m white square.

Left cuts the chocolate bar into two parts, and she eats the part with fewer than or equal to $\lceil \frac{n}{2} \rceil$ black blocks. When there is only a one-by-one white chocolate bar in figures 5, she can eat it.

Similarly, Right cuts the chocolate bar into two and eats the part with fewer than or equal to $\lceil \frac{m}{2} \rceil$ white blocks. When there is only a one-by-one black chocolate bar in figures 5, he can eat it.

A player loses in the game when she or he cannot eat the remaining chocolate bar.

Here, we have four outcome classes.
(a) A position is called a $\mathcal{P}$-*position* if it is a winning position for the previous player (the player who just moved), as long as he/she plays correctly at every stage.
(b) A position is called an $\mathcal{N}$-*position* if it is a winning position for the next player, as long as he/she plays correctly at every stage.
(c) A position is referred to as a $\mathcal{L}$-*position* if it is a winning position for L, as long as she plays correctly at every stage.
(d) A position is referred to as an $\mathcal{R}$-*position* if it is a winning position for R, as long as he plays correctly at every stage.

Theorem 1. *Every position of a game belongs to exactly one of four outcome classes $\mathcal{P}$-position, $\mathcal{N}$-position, $\mathcal{L}$-position, and $\mathcal{R}$-position.*

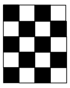

Figure 1:(5,4,1)

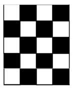

Figure 2: (5,4,0)

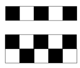

Figure 3:

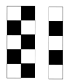

Figure 4:

Figure 5:

We denote a chocolate bar by (x, y, s), where x and y are the height and the width of the chocolate bar, respectively, and s is 1 if the square on the upper left corner is black and 0 if it is white.

According to the rule of the game, there are three ways that the game ends. A player loses the game when there is no chocolate bar left. Player L loses when there is only 1×1 black square, and Player R loses when there is only 1×1 white square.

Definition 1. *Let* $\mathcal{P}_a = \{(2p, 2^n(p+1)-2, s) : n, p \in \mathbb{N}, s = 0, 1\}$*,* $\mathcal{P}_b = \{(2^n(p+1)-2, 2p, s) : n, p \in \mathbb{N}, s = 0, 1\}$ *and* $\mathcal{P}_c = \{(2p+1, 2p+1, s) : p \in \mathbb{N}, s = 0, 1\}$*. Let* $\mathcal{P} = \mathcal{P}_a \cup \mathcal{P}_b \cup \mathcal{P}_c$*.*

Definition 2. *Let* $\mathcal{L}_a = \{(2p+1, 2q+1, 1) : p, q \in \mathbb{N}, q \geq 3\}$*,* $\mathcal{L}_b = \{(2p+1, 2q+1, 0) : p, q \in \mathbb{N}, p \geq 3\}$*,* $\mathcal{L}_c = \{(1, 2p+1, 0) : p \in \mathbb{N}\}$*, and*
$\mathcal{L}_d = \{(2p+1, 1, 0) : p \in \mathbb{N}\}$*. Let* $\mathcal{L} = \mathcal{L}_a \cup \mathcal{L}_b \cup \mathcal{L}_c \cup \mathcal{L}_d$*.*

Theorem 2. *We have the following* (i) *and* (ii)*:*
(i) *The set $\mathcal{P}$ is the set of $\mathcal{P}$-positions.*
(ii) *The set $\mathcal{L}$ is the set of $\mathcal{L}$-positions.*

Note that we can get the set of $\mathcal{R}$-positions by substituting black and white squares in $\mathcal{L}$, and the complement of $\mathcal{P} \cup \mathcal{L} \cup \mathcal{R}$ is $\mathcal{N}$.

References

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Comply/constrain operator of combinatorial games

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Abstract

We consider a comply/constrain operator on impartial rulesets. Applied to the rulesets A and B, on each turn, the opponent proposes one of the rulesets and the current player complies, by playing a move in that ruleset. If the outcome table of the comply/constrain variation of A and B is the same as the outcome table of A, then we say that B is dominated by A. We show necessary and sufficient conditions of "A dominates B" and some properties on comply/constrain operator. This study is a continuation of [1].

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A DIRECT PROOF OF SOKOBAN∈IP

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Abstract

SOKOBAN is a representative puzzle in the class of PSPACE-complete problems. The upper bound for the complexity of SOKOBAN is straight forward, as it is easy to see SOKOBAN∈NPSPACE, while NPSPACE=PSPACE. For the lower bound, there are at least three quite different proofs of the PSPACE-hardness of SOKOBAN[1, 2, 4]. One of the most remakable theorems in computational complexity theory is that IP=PSPACE [5]. However, it is not easy to show directly that SOKOBAN∈IP. Note that telling the verifier a complete solution to a SOKOBAN level doesn't work, because the solution might be of exponential length which cannot be verified in polynomial time. The proof method of IP=PSPACE is not readily applied to SOKOBAN. The standard proof [6] of IP=PSPACE is to show that a PSPACE-complete problem, TQBF, is in IP. Another proof [3] shows that any PSPACE language accepted by a deterministic Turing Machine has an interactive proof. Both proofs base on the arithmetization technique. SOKOBAN is inherently non-deterministic in nature. In this talk, we extend the arithmetization technique to non-deterministic case by showing that SOKOBAN is in IP. This gives a concrete way of convincing the verifier that a SOKOBAN level is solvable, but without telling the complete solution to that level.

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8	Awanis	Zata Yumni	Universitas Mataram	Indonesia
9	Aziz	Ali	University of Southampton	United Kingdom
10	Barish	Robert	The University of Tokyo	Japan
11	Baskoro	Edy Tri	Institut Teknologi Bandung	Indonesia
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13	Bustan	Ariestha	Institut Teknologi Bandung	Indonesia
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30	Horiyama	Takashi	Hokkaido University	Japan
31	Ikeyama	Airi	Nagoya University	Japan
32	Imrona	Mahmud	Institut Teknologi Bandung	Indonesia
33	Inazu	Hiroki	Hiroshima University	Japan
34	Itoh	Jin-ichi	Sugiyama Jogakuen University	Japan
35	John	Peter	Universitas Indonesia	Indonesia
36	Kamata	Tonan	JAIST	Japan

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38	KODATE	Takako	Tokyo Woman's Christian University	Japan
39	korivand	meysam	Alzahra uni	iran
40	Krishnakumar	Aditi	Open University	India
41	Kruft Welton	Eartha	Open University	United Kindom
42	Kuwata	Takayasu	Tokai University	Japan
43	Lewchalermvongs	Chanun	Mahidol University	Thailand
44	Liu	Hong	Institute for Basic Science	South Korea
45	Lynch	Jayson	Massachusetts Institute of Technology	USA
46	Maryati	Tita Khalis	UIN Syarif Hidayatullah Jakarta	Indonesia
47	Matsui	Yasuko	Tokai University	Japan
48	Maulana	hendri	Institut Teknologi Bandung	Indonesia
49	Miyadera	Ryohei	Keimei Gakuin	Japan
50	Mizugaki	Keita	Nishinomiya High School	Japan
51	Nabila	Maya	Institut Teknologi Bandung	Indonesia
52	Nakajima	Chihiro	Faculty of engineering, Tohoku Bunka Gakuen University	Japan
53	Nara	Chie	Meiji University	Japan
54	Ngurah	Anak Agung Gede	Universitas Merdeka Malang	Indonesia
55	Noro	Kohei	Nagoya University	Japan
56	Obata	Nobuaki	Tohoku University	Japan
57	Oktariani	Finny	Institut Teknologi Bandung	Indonesia
58	Ono	Hirotaka	Nagoya University	Japan
59	Pancahayani	Sigit	Institut Teknologi Bandung	Indonesia
60	Putri	Niluh Putu Aprillia Pu	Tadulako University	Indonesia
61	Putri	Pritta Etriana	Institut Teknologi Bandung	Indonesia
62	Rahadi	Andi	Institut Teknologi Bandung	Indonesia
63	Rahmadani	Desi	Universitas Negeri Malang	Indonesia
64	Ramdani	Rismawati	UIN Sunan Gunung Djati Bandung	Indonesia
65	Ruangwises	Suthee	The University of Electro-Communications	Japan
66	Sakai	Toshinori	Tokai University	Japan
67	Santika	Aditya Purwa	Bandung Institute of Technology	Indonesia
68	Saputro	Suhadi Wido	Institut Teknologi Bandung	Indonesia
69	Sherlin	Intan	Institut Teknologi Bandung	Indonesia
70	Shiota	Takumi	Kyushu Institute of Technology	Japan
71	Shulhany	Ahmad	Institut Teknologi Bandung	Indonesia
72	Silaban	Denny	Universitas Indonesia	Indonesia

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74	Sugeng	Kiki	Universitas Indonesia	Indonesia
75	Supanut	Chaidee	Chiang Mai University	Thailand
76	Susanto	Faisal	Institut Teknologi Bandung	Indonesia
77	Suwastika	Erma	Institut Teknologi Bandung	Indonesia
78	Takahashi	Shoei	Keimei Gakuin	Japan
79	Tan	Xuehou	Tokai University	Japan
80	Tolentino	Mark Anthony	Ateneo de Manila University	Philippines
81	Tomoaki	Abuku	National Institute of Informatics	Japan
82	Tsutsumi	Kotaro	University of Tsukuba	Japan
83	Tuite	James	Open University	United Kingdom
84	Uehara	Ryuhei	Japan Advanced Institute of Science and Technology	Japan
85	Umbara	Rian Febrian	Institut Teknologi Bandung	Indonesia
86	Wanditra	Lucky Cahya	Institut Teknologi Bandung	Indonesia
87	Wannasit	Wannasiri	Chiang Mai University	Thailand
88	Wulandari	Risma Yulina	Bandung Institute of Technology	Indonesia
89	Xuehou	Tan	Tokai University	Japan
90	Yamada	Suguru	Kyushu University	Japan
91	Yamashita	Takahiro	Hiroshima University	Japan
92	Yang	Chao	Guangdong University of Foreign Studies	China
93	Zakiyyah	Alfi Yusrotis	Bina Nusantara University	Indonesia